Homework-7

44. ***REASONING*** The magnetic moment of a current-carrying coil is discussed in Section 21.6, where it is given as

$$\text{Magnetic moment} = NIA \tag{1}$$

In Equation (1), N is the number of turns in the coil, I is the current it carries, and A is its area. Both coils in this problem are circular, so their areas are calculated from their radii via $A = \pi r^2$.

SOLUTION Because the magnetic moments of the two coils are equal, Equation (1) yields

$$N_2 I_2 A_2 = N_1 I_1 A_1 \tag{2}$$

Substituting $A = \pi r^2$ into Equation (2) and solving for r_2, we obtain

$$N_2 I_2 \left(\cancel{\pi} r_2^2\right) = N_1 I_1 \left(\cancel{\pi} r_1^2\right) \quad \text{or} \quad r_2^2 = r_1^2 \left(\frac{N_1 I_1}{N_2 I_2}\right) \quad \text{or} \quad r_2 = r_1 \sqrt{\frac{N_1 I_1}{N_2 I_2}} \tag{3}$$

Therefore, the radius of the second coil is

$$r_2 = r_1 \sqrt{\frac{N_1 I_1}{N_2 I_2}} = (0.088\ \text{m}) \sqrt{\frac{(140)(4.2\ \text{A})}{(170)(9.5\text{A})}} = \boxed{0.053\ \text{m}}$$

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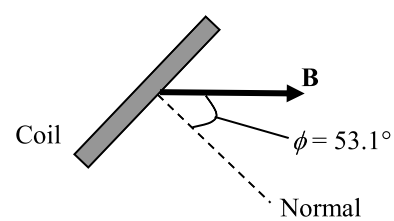
46. ***REASONING AND SOLUTION*** The torque is given by $\tau = NIAB \sin \phi$.

a. The maximum torque occurs when $\phi = 90.0°$ ($\sin \phi = 1$). In this case we want the torque to be 80.0% of the maximum value, so

$$NIAB \sin \phi = 0.800\,(NIAB \sin 90.0°) \quad \text{so that} \quad \phi = \sin^{-1}(0.800) = \boxed{53.1°}$$

b. The edge-on view of the coil at the right shows the normal to the plane of the coil, the magnetic field **B**, and the angle ϕ.



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53. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION***

a. In Figure 21.26a the magnetic field that exists at the location of each wire points upward. Since the current in each wire is the same, the fields at the locations of the wires also have the same magnitudes. Therefore, a single external field that points $\boxed{\text{downward}}$ will cancel the mutual repulsion of the wires, if this external field has a magnitude that equals that of the field produced by either wire.

b. Equation 21.5 gives the magnitude of the field produced by a long straight wire. The external field must have this magnitude:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(25\ \text{A})}{2\pi(0.016\ \text{m})} = \boxed{3.1\times 10^{-4}\ \text{T}}$$

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54. ***REASONING*** The torque τ exerted on a coil with a current I_{coil} is given by $\tau = NI_{\text{coil}}AB\sin\phi$ (Equation 21.4), where N is the number of turns in the coil, A is its area, B is the magnitude of the external magnetic field causing the torque, and ϕ is the angle between the normal to the surface of the coil and the direction of the external magnetic field. The external magnetic field B, in this case, is the magnetic field of the solenoid. We will use $B = \mu_0 nI$ (Equation 21.7) to determine the magnetic field, where $\mu_0 = 4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}$ is the permeability of free space, n is the number of turns per meter of length of the solenoid, and I is the current in the solenoid. The magnetic field in the interior of a solenoid is parallel to the solenoid's axis. Because the normal to the surface of the coil is perpendicular to the axis of the solenoid, the angle ϕ is equal to 90.0°.

SOLUTION Substituting the expression $B = \mu_0 nI$ (Equation 21.7) for the magnetic field of the solenoid into $\tau = NI_{\text{coil}}AB\sin\phi$ (Equation 21.4), we obtain

$$\tau = NI_{\text{coil}}AB\sin\phi = NI_{\text{coil}}A\left(\mu_0 nI\right)\sin\phi = \mu_0 nNAI_{\text{coil}}I\sin\phi$$

Therefore, the torque exerted on the coil is

$$\tau = \left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1400\ \text{m}^{-1}\right)(50)\left(1.2\times 10^{-3}\ \text{m}^2\right)(0.50\ \text{A})(3.5\ \text{A})\sin 90.0^\circ$$

$$= \boxed{1.8\times 10^{-4}\ \text{N}\cdot\text{m}}$$

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56. ***REASONING AND SOLUTION*** The magnetic field at the center of a current loop of radius R is given by $B = \mu_0 I/(2R)$, so that

$$R = \frac{\mu_0 I}{2B} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(12\text{ A})}{2\left(1.8\times10^{-4}\,\text{T}\right)} = \boxed{4.2\times10^{-2}\text{ m}}$$

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59. ***REASONING AND SOLUTION*** Let the current in the left-hand wire be labeled I_1 and that in the right-hand wire I_2.

a. At point A: B_1 is *up* and B_2 is *down*, so we subtract them to get the net field. We have

$$B_1 = \mu_0 I_1/(2\pi d_1) = \mu_0(8.0\text{ A})/[2\pi(0.030\text{ m})]$$
$$B_2 = \mu_0 I_2/(2\pi d_2) = \mu_0(8.0\text{ A})/[2\pi(0.150\text{ m})]$$

So the net field at point A is

$$B_A = B_1 - B_2 = \boxed{4.3\times10^{-5}\text{ T}}$$

b. At point B: B_1 and B_2 are both *down* so we add the two. We have

$$B_1 = \mu_0(8.0\text{ A})/[2\pi(0.060\text{ m})]$$
$$B_2 = \mu_0(8.0\text{ A})/[2\pi(0.060\text{ m})]$$

So the net field at point B is

$$B_B = B_1 + B_2 = \boxed{5.3\times10^{-5}\text{ T}}$$

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68. ***REASONING*** Since the two wires are next to each other, the net magnetic field is everywhere parallel to $\Delta\ell$ in Figure 21.37. Moreover, the net magnetic field **B** has the same magnitude B at each point along the circular path, because each point is at the same distance from the wires. Thus, in Ampère's law (Equation 21.8), $B_{\|} = B$, $I = I_1 + I_2$, and we have

$$\Sigma B_{\|}\Delta\ell = B(\Sigma\Delta\ell) = \mu_0\left(I_1 + I_2\right)$$

But $\Sigma\Delta\ell$ is just the circumference ($2\pi r$) of the circle, so Ampère's law becomes

$$B(2\pi r) = \mu_0\left(I_1 + I_2\right)$$

This expression can be solved for *B*.

SOLUTION

a. When the currents are in the same direction, we find that

$$B=\frac{\mu_0\left(I_1+I_2\right)}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(28\ \text{A}+12\ \text{A}\right)}{2\pi\left(0.72\ \text{m}\right)}=\boxed{1.1\times10^{-5}\ \text{T}}$$

b. When the currents have opposite directions, a similar calculation shows that

$$B=\frac{\mu_0\left(I_1-I_2\right)}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(28\ \text{A}-12\ \text{A}\right)}{2\pi\left(0.72\ \text{m}\right)}=\boxed{4.4\times10^{-6}\ \text{T}}$$