Homework #5

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80. ***REASONING*** In part *a* of the drawing in the text, the current goes from left-to-right through the resistor. Since the current always goes from a higher to a lower potential, the left end of the resistor is + and the right end is –. In part *b*, the current goes from right-to-left through the resistor. The right end of the resistor is + and the left end is –. The potential drops and rises for the two cases are:

	Potential drops	Potential rises
Part *a*	*IR*	*V*
Part *b*	*V*	*IR*

SOLUTION Since the current *I* goes from left-to-right through the 3.0-Ω and 4.0-Ω resistors, the left end of each resistor is + and the right end is –. The current goes through the 5.0-Ω resistor from right-to-left, so the right end is + and the left end is –. Starting at the upper left corner of the circuit, and proceeding clockwise around it, Kirchhoff's loop rule is written as

$$\underbrace{(3.0\ \Omega)I + 12\ \text{V} + (4.0\ \Omega)I + (5.0\ \Omega)I}_{\text{Potential drops}} = \underset{\substack{\text{Potential} \\ \text{rises}}}{36\ \text{V}}$$

Solving this equation for the current gives $I = \boxed{2.0\ \text{A}}$

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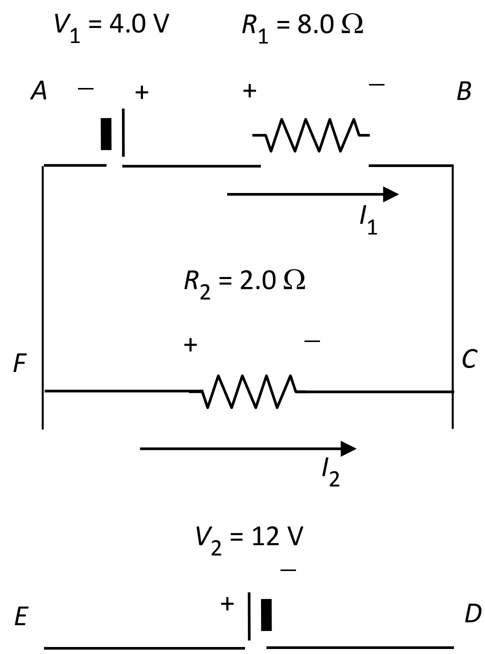
83. ***REASONING*** This problem can be solved by using Kirchhoff's loop rule. We begin by drawing a current through each resistor. The drawing shows the directions chosen for the currents. The directions are arbitrary, and if any one of them is incorrect, then the analysis will show that the corresponding value for the current is negative.



We mark the two ends of each resistor with plus and minus signs that serve as an aid in identifying the potential drops and rises for the loop rule, recalling that conventional current is always directed from a higher potential (+) toward a lower potential (–). Thus, given the directions chosen for I_1 and I_2, the plus and minus signs *must* be those shown in the drawing. We then apply Kirchhoff's loop rule to the top loop (*ABCF*) and to the bottom loop (*FCDE*) to determine values for the currents I_1 and I_2.

SOLUTION Applying Kirchhoff's loop rule to the top loop (*ABCF*) gives

$$\underbrace{V_1 + I_2 R_2}_{\text{Potential rises}} = \underbrace{I_1 R_1}_{\text{Potential drop}} \quad (1)$$

Similarly, for the bottom loop (*FCDE*),

$$\underbrace{V_2}_{\text{Potential rise}} = \underbrace{I_2 R_2}_{\text{Potential drop}} \quad (2)$$

Solving Equation (2) for I_2 gives

$$I_2 = \frac{V_2}{R_2} = \frac{12\ \text{V}}{2.0\ \Omega} = \boxed{6.0\ \text{A}}$$

Since I_2 is a positive number, the current in the resistor R_2 goes from $\boxed{\text{left to right}}$, as shown in the drawing. Solving Equation (1) for I_1 and substituting $I_2 = V_2/R_2$ into the resulting expression yields

$$I_1 = \frac{V_1 + I_2 R_2}{R_1} = \frac{V_1 + \left(\dfrac{V_2}{R_2}\right) R_2}{R_1} = \frac{V_1 + V_2}{R_1} = \frac{4.0\ \text{V} + 12\text{V}}{8.0\ \Omega} = \boxed{2.0\ \text{A}}$$

Since I_1 is a positive number, the current in the resistor R_1 goes from $\boxed{\text{left to right}}$, as shown in the drawing.

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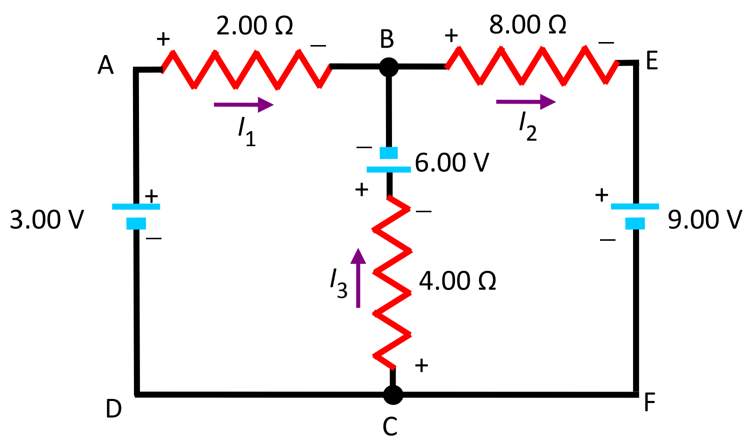
84. ***REASONING*** In preparation for applying Kirchhoff's rules, we now choose the currents in each resistor. The directions of the currents are arbitrary, and should they be incorrect, the currents will turn out to be negative quantities. Having chosen the currents, we also mark the ends of the resistors with the plus and minus signs that indicate that the currents are directed from higher (+) toward lower (–) potential. These plus and minus signs will guide us when we apply Kirchhoff's loop rule.



SOLUTION Applying the junction rule to junction B, we find

$$\underbrace{I_1 + I_3}_{\text{Into junction}} = \underbrace{I_2}_{\text{Out of junction}} \tag{1}$$

Applying the loop rule to loop ABCD (going clockwise around the loop), we obtain

$$\underbrace{I_1(2.00\ \Omega)}_{\text{Potential drops}} = \underbrace{6.00\ \text{V} + I_3(4.00\ \Omega) + 3.00\ \text{V}}_{\text{Potential rises}} \tag{2}$$

Applying the loop rule to loop BEFC (going clockwise around the loop), we obtain

$$\underbrace{I_2(8.00\ \Omega) + 9.00\ \text{V} + I_3(4.00\ \Omega) + 6.00\ \text{V}}_{\text{Potential drops}} = \underbrace{0}_{\text{Potential rises}} \tag{3}$$

Substituting I_2 from Equation (1) into Equation (3) gives

$$\left(I_1+I_3\right)(8.00\ \Omega)+9.00\ \text{V}+I_3(4.00\ \Omega)+6.00\ \text{V}=0$$

$$I_1(8.00\ \Omega)+I_3(12.00\ \Omega)+15.00\ \text{V}=0 \qquad (4)$$

Solving Equation (2) for I_1 gives

$$I_1=4.50\ \text{A}+I_3(2.00)$$

This result may be substituted into Equation (4) to show that

$$\left[4.50\ \text{A}+I_3(2.00)\right](8.00\ \Omega)+I_3(12.00\ \Omega)+15.00\ \text{V}=0$$

$$I_3(28.00\ \Omega)+51.00\ \text{V}=0 \quad \text{or} \quad I_3=\frac{-51.00\ \text{V}}{28.00\ \Omega}=\boxed{-1.82\ \text{A}}$$

The minus sign indicates that $\boxed{\text{the current in the 4.00-}\Omega\text{ resistor is directed downward}}$, rather than upward as selected arbitrarily in the drawing.

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94. ***REASONING*** The equivalent capacitance C_S of a set of three capacitors connected in series is given by $\frac{1}{C_S}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$ (Equation 20.19). In this case, we know that the equivalent capacitance is C_S = 3.00 μF, and the capacitances of two of the individual capacitors in this series combination are C_1 = 6.00 μF and C_2 = 9.00 μF. We will use Equation 20.19 to determine the remaining capacitance C_3.

SOLUTION Solving Equation 20.19 for C_3, we obtain

$$\frac{1}{C_3} = \frac{1}{C_S} - \frac{1}{C_1} - \frac{1}{C_2} \qquad \text{or} \qquad C_3 = \frac{1}{\dfrac{1}{C_S} - \dfrac{1}{C_1} - \dfrac{1}{C_2}}$$

Therefore, the third capacitance is

$$C_3 = \frac{1}{\dfrac{1}{3.00\ \mu\text{F}} - \dfrac{1}{6.00\ \mu\text{F}} - \dfrac{1}{9.00\ \mu\text{F}}} = \boxed{18\ \mu\text{F}}$$

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97. ***REASONING*** Our approach to this problem is to deal with the arrangement in parts. We will combine separately those parts that involve a series connection and those that involve a parallel connection.

SOLUTION The 24, 12, and 8.0-μF capacitors are in series. Using Equation 20.19, we can find the equivalent capacitance for the three capacitors:

$$\frac{1}{C_s} = \frac{1}{24\ \mu\text{F}} + \frac{1}{12\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} \qquad \text{or} \qquad C_s = 4.0\ \mu\text{F}$$

This 4.0-μF capacitance is in parallel with the 4.0-μF capacitance already shown in the text diagram. Using Equation 20.18, we find that the equivalent capacitance for the parallel group is

$$C_p = 4.0\ \mu\text{F} + 4.0\ \mu\text{F} = 8.0\ \mu\text{F}$$

This 8.0-μF capacitance is between the 5.0 and the 6.0-μF capacitances and in series with them. Equation 20.19 can be used, then, to determine the equivalent capacitance between *A* and *B* in the text diagram:

$$\frac{1}{C_s} = \frac{1}{5.0\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} + \frac{1}{6.0\ \mu\text{F}} \qquad \text{or} \qquad C_s = \boxed{2.0\ \mu\text{F}}$$

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99. ***REASONING***

a. When capacitors are wired in parallel, the total charge q supplied to them is the sum of the charges supplied to the individual capacitors, or $q = q_1 + q_2$. The individual charges can be obtained from $q_1 = C_1V$ and $q_2 = C_2V$, since the capacitances, C_1 and C_2, and the voltage V are known.

b. When capacitors are wired in series, the voltage V across them is equal to the sum of the voltages across the individual capacitors, or $V = V_1 + V_2$. However, the charge q on each capacitor is the same. The individual voltages can be obtained from $V_1 = q/C_1$ and $V_2 = q/C_2$.

SOLUTION

a. Substituting $q_1 = C_1V$ and $q_2 = C_2V$ (Equation 19.8) into $q = q_1 + q_2$, we have

$$q = q_1 + q_2 = C_1V + C_2V = (C_1 + C_2)V$$

$$= (2.00\times10^{-6}\ \text{F} + 4.00\times10^{-6}\ \text{F})(60.0\ \text{V}) = \boxed{3.60\times10^{-4}\ \text{C}}$$

b. Substituting $V_1 = q/C_1$ and $V_2 = q/C_2$ (Equation 19.8) into $V = V_1 + V_2$ gives

$$V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2}$$

Solving this relation for q, we have

$$q=\frac{V}{\frac{1}{C_1}+\frac{1}{C_2}}=\frac{60.0\text{ V}}{\frac{1}{2.00\times10^{-6}\text{ F}}+\frac{1}{4.00\times10^{-6}\text{ F}}}=\boxed{8.00\times10^{-5}\text{ C}}$$

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100. ***REASONING AND SOLUTION*** The 7.00 and 3.00-μF capacitors are in parallel. According to Equation 20.18, the equivalent capacitance of the two is 7.00 μF + 3.00 μF = 10.0 μF. This 10.0-μF capacitance is in series with the 5.00-μF capacitance. According to Equation 20.19, the equivalent capacitance of the complete arrangement can be obtained as follows:

$$\frac{1}{C}=\frac{1}{10.0\ \mu\text{F}}+\frac{1}{5.00\ \mu\text{F}}=0.300\left(\mu\text{F}\right)^{-1} \qquad \text{or} \qquad C=\frac{1}{0.300\left(\mu\text{F}\right)^{-1}}=3.33\ \mu\text{F}$$

The battery separates an amount of charge

$$Q=CV=(3.33\times10^{-6}\text{ F})(30.0\text{ V})=99.9\times10^{-6}\text{ C}$$

This amount of charge resides on the 5.00 μF capacitor, so its voltage is

$$V_5=(99.9\times10^{-6}\text{ C})/(5.00\times10^{-6}\text{ F})=20.0\text{ V}$$

The loop rule gives the voltage across the 3.00 μF capacitor to be

$$V_3=30.0\text{ V}-20.0\text{ V}=10.0\text{ V}$$

This is also the voltage across the 7.00 μF capacitor, since it is in parallel, so V_7 = $\boxed{10.0\text{ V}}$.
