Homework#2

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42. ***REASONING AND SOLUTION*** The magnitude of the force on q_1 due to q_2 is given by Coulomb's law:

$$F_{12} = \frac{k|q_1||q_2|}{r_{12}^{\;2}} \qquad (1)$$

The magnitude of the force on q_1 due to the electric field of the capacitor is given by

$$F_{1C} = |q_1|E_C = |q_1|\left(\frac{\sigma}{\varepsilon_0}\right) \qquad (2)$$

Equating the right hand sides of Equations (1) and (2) above gives

$$\frac{k|q_1||q_2|}{r_{12}^{\;2}} = |q_1|\left(\frac{\sigma}{\varepsilon_0}\right)$$

Solving for r_{12} gives

$$r_{12} = \sqrt{\frac{\varepsilon_0 k|q_2|}{\sigma}}$$

$$= \sqrt{\frac{[8.85\times10^{-12}\ \mathrm{C^2/(N\cdot m^2)}](8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(5.00\times10^{-6}\ \mathrm{C})}{(1.30\times10^{-4}\ \mathrm{C/m^2})}} = \boxed{5.53\times10^{-2}\ \mathrm{m}}$$

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46. ***REASONING AND SOLUTION*** From kinematics, $v_y^2 = v_{0y}^2 + 2a_y y$. Since the electron starts from rest, v_{0y} = 0 m/s. The acceleration of the electron is given by

$$a_y = \frac{F}{m} = \frac{eE}{m}$$

where *e* and *m* are the electron's charge magnitude and mass, respectively, and *E* is the magnitude of the electric field. The magnitude of the electric field between the plates of a parallel plate capacitor is $E = \sigma/\varepsilon_0$, where σ is the magnitude of the charge per unit area on each plate. Thus, $a_y = e\sigma/(m\varepsilon_0)$. Combining this expression for a with the kinematics equation we have

$$v_y^2 = 2\left(\frac{e\sigma}{m\varepsilon_0}\right)y$$

Solving for v_y gives

$$v_y = \sqrt{\frac{2e\sigma y}{m\varepsilon_0}} = \sqrt{\frac{2\left(1.60\times10^{-19}\ \text{C}\right)\left(1.8\times10^{-7}\ \text{C/m}^2\right)\left(1.5\times10^{-2}\ \text{m}\right)}{\left(9.11\times10^{-31}\ \text{kg}\right)\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right]}} = \boxed{1.0\times10^{7}\ \text{m/s}}$$

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54. ***REASONING AND SOLUTION*** Gauss' Law is given by text Equation 18.7: $\Phi_E = \dfrac{Q}{\varepsilon_0}$, where *Q* is the *net charge* enclosed by the Gaussian surface.

a. $$\Phi_E = \frac{3.5\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = \boxed{4.0\times10^{5}\ \text{N}\cdot\text{m}^2/\text{C}}$$

b. $$\Phi_E = \frac{-2.3\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = \boxed{-2.6\times10^{5}\ \text{N}\cdot\text{m}^2/\text{C}}$$

c. $\Phi_E = \dfrac{(3.5\times10^{-6}\ \text{C})+(-2.3\times10^{-6}\ \text{C})}{8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = \boxed{1.4\times10^{5}\ \text{N}\cdot\text{m}^2/\text{C}}$

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58. ***REASONING*** The charge *Q* inside the rectangular box is related to the electric flux Φ_E that passes through the surfaces of the box by Gauss' law, $Q=\varepsilon_0\Phi_E$ (Equation 18.7), where ε_0 is the permittivity of free space. The electric flux is the algebraic sum of the flux through each of the six surfaces.

SOLUTION The charge inside the box is

$$Q=\varepsilon_0\Phi_E=\varepsilon_0\left(\Phi_1+\Phi_2+\Phi_3+\Phi_4+\Phi_5+\Phi_6\right)$$

$$=\left[8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right]\left(+1500\frac{\text{N}\cdot\text{m}^2}{\text{C}}+2200\frac{\text{N}\cdot\text{m}^2}{\text{C}}+4600\frac{\text{N}\cdot\text{m}^2}{\text{C}}\right.$$

$$\left.-1800\frac{\text{N}\cdot\text{m}^2}{\text{C}}-3500\frac{\text{N}\cdot\text{m}^2}{\text{C}}-5400\frac{\text{N}\cdot\text{m}^2}{\text{C}}\right)$$

$$=\boxed{-2.1\times10^{-8}\ \text{C}}$$

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60. ***REASONING*** Gauss' Law, $\Sigma\left(E\cos\phi\right)\Delta A=\dfrac{Q}{\varepsilon_0}$ (Equation 18.7), relates the electric field magnitude *E* on a Gaussian surface to the net charge *Q* enclosed by that surface; $\Phi_E=\Sigma\left(E\cos\phi\right)\Delta A$ (Equation 18.6) is the electric flux through the Gaussian surface (divided into many tiny sections of area Δ*A*) and ε_0 is the permittivity of free space. We are to determine the magnitude *E* of the electric field due to electric charges that are spread uniformly over the surfaces of two concentric

spherical shells. The electric field due to these charges possesses spherical symmetry, so we will choose Gaussian surfaces in the shape of spheres concentric with the shells. The radius *r* of each Gaussian surface will be equal to the distance from the common center of the shells and will be the distance at which we are to evaluate the electric field.

Because the electric field has spherical symmetry, the magnitude *E* of the electric field is constant at all points on any such spherical Gaussian surface. Furthermore, the electric field is directed either radially outward (if the net charge within the Gaussian surface is positive) or radially inward (if the net charge within the Gaussian surface is negative). This means that the angle ϕ between the electric field and the normal to any such spherical Gaussian surface is either 0.0° or 180°. The quantity $E\cos\phi$, therefore, is constant, and may be factored out of the summation in Equation 18.6:

$$\Phi_{\mathrm{E}} = \Sigma(E\cos\phi)\Delta A = (E\cos\phi)\Sigma\Delta A \tag{1}$$

The sum $\Sigma\Delta A$ of all the tiny sections of area ΔA that compose a spherical Gaussian surface is the total surface area $\Sigma\Delta A = A = 4\pi r^2$ of a sphere of radius *r*. Thus, Equation (1) becomes

$$\Phi_{\mathrm{E}} = (E\cos\phi)\Sigma\Delta A = 4\pi r^2 E\cos\phi \tag{2}$$

SOLUTION

a. The outer shell has a radius r_2 = 0.15 m, and we are to determine the electric field at a distance r = 0.20 m from the common center of the shells. Therefore, we will choose a spherical Gaussian surface (radius r = 0.20 m) that encloses both shells and shares their common center. According to Gauss' law and Equation (2), we have that the net electric flux Φ_{E} through this sphere is

$$\Sigma(E\cos\phi)\Delta A = 4\pi r^2 E\cos\phi = \frac{Q}{\varepsilon_0} \tag{3}$$

Solving Equation (3) for *E* yields

$$E = \frac{Q}{4\pi\varepsilon_0 r^2\cos\phi} \tag{4}$$

Because the chosen Gaussian surface encloses both shells, the net charge *Q* enclosed by the surface is $Q = q_1 + q_2$. The positive charge q_2 on the outer shell has a larger magnitude than the negative charge q_1 on the inner shell, so that *Q* is a positive net charge. Therefore, the electric field is directed $\boxed{\text{radially outward}}$, and the angle between the electric field and the normal to the surface of the spherical Gaussian surface is $\phi = 0.0°$. Therefore, Equation (4) gives the electric field magnitude as

$$E = \frac{Q}{4\pi\varepsilon_0 r^2 \cos\phi} = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2 \cos\phi} = \frac{-1.6\times10^{-6}\ \text{C} + 5.1\times10^{-6}\ \text{C}}{4\pi\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.20\ \text{m})^2 \cos 0.0°}$$

$$= \boxed{7.9\times10^{5}\ \text{N/C}}$$

b. We again choose a spherical Gaussian surface concentric with the shells, this time of radius $r = 0.10$ m. The radius of this sphere is greater than the radius ($r_1 = 0.050$ m) of the inner shell but less than the radius ($r_2 = 0.15$ m) of the outer shell. Therefore, this Gaussian surface is located *between* the two shells and encloses only the charge on the inner shell: $Q = q_1$. This is a negative charge, so that the electric field is directed $\boxed{\text{radially inward}}$, and the angle between the electric field and the normal to the surface of the Gaussian sphere is $\phi = 180°$. From Equation (4), then, we have that

$$E = \frac{Q}{4\pi\varepsilon_0 r^2 \cos\phi} = \frac{q_1}{4\pi\varepsilon_0 r^2 \cos\phi} = \frac{-1.6\times10^{-6}\ \text{C}}{4\pi\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.10\ \text{m})^2 \cos 180°}$$

$$= \boxed{1.4\times10^{6}\ \text{N/C}}$$

c. Choosing a spherical Gaussian surface with a radius of $r = 0.025$ m, we see that it is entirely inside the inner shell ($r_1 = 0.050$ m). Therefore, the enclosed charge is zero: $Q = 0$ C. Equation (4) shows that the electric field at this distance from the common center is zero:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2 \cos\phi} = \frac{0\ \text{C}}{4\pi\varepsilon_0 r^2 \cos\phi} = \boxed{0\ \text{N/C}}$$

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2. ***REASONING AND SOLUTION*** The only force that acts on the α-particle is the conservative electric force. Therefore, the total energy of the α-particle is conserved as it moves from point *A* to point *B*:

$$\underbrace{\tfrac{1}{2}mv_A^2 + \text{EPE}_A}_{\text{Total energy at point } A} = \underbrace{\tfrac{1}{2}mv_B^2 + \text{EPE}_B}_{\text{Total energy at point } B}$$

Since the α-particle starts from rest, v_A = 0 m/s. The electric potential V is related to the electric potential energy EPE by V = EPE/q (see Equation 19.3). With these changes, the equation above gives the kinetic energy of the α-particle at point B to be

$$\tfrac{1}{2}mv_B^2 = \text{EPE}_A - \text{EPE}_B = q\left(V_A - V_B\right)$$

Since an α-particle contains two protons, its charge is q = 2e = 3.2 × 10^{-19} C. Thus, the kinetic energy (in electron-volts) is

$$\tfrac{1}{2}mv_B^2 = q\left(V_A - V_B\right) = \left(3.2\times10^{-19}\ \text{C}\right)\left[+250\ \text{V} - \left(-150\ \text{V}\right)\right]$$

$$= 1.28\times10^{-16}\ \text{J}\left(\frac{1.0\ \text{eV}}{1.6\times10^{-19}\ \text{J}}\right) = \boxed{8.0\times10^{2}\ \text{eV}}$$

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9. ***REASONING***

a. The work W_{AB} done by the electric force in moving a charge q from A to B is related to the potential difference $V_A - V_B$ between the two points by Equation 19.4 as $W_{AB} = q(V_A - V_B)$. Letting A = ground and B = cloud, the work done can be written as

$$W_{\text{ground-cloud}} = q(V_{\text{ground}} - V_{\text{cloud}}) = -q(V_{\text{cloud}} - V_{\text{ground}}) \qquad (1)$$

b. According to the work-energy theorem (Equation 6.3), the work W done on an object of mass m is equal to its final kinetic energy $\tfrac{1}{2}mv_{\text{f}}^2$ minus its initial kinetic energy $\tfrac{1}{2}mv_0^2$. Setting $W = W_{\text{ground-cloud}}$ and noting that v_0 = 0 m/s, since the automobile starts from rest, the work-energy theorem takes the form $W_{\text{ground-cloud}} = \tfrac{1}{2}mv_{\text{f}}^2$. Solving this equation for v_{f} and substituting Equation (1) for $W_{\text{ground-cloud}}$, the final speed of the car is

$$v_{\text{f}} = \sqrt{\frac{2W_{\text{ground-cloud}}}{m}} = \sqrt{\frac{-2q\left(V_{\text{cloud}} - V_{\text{ground}}\right)}{m}} \qquad (2)$$

c. If the work $W_{\text{ground-cloud}}$ were converted completely into heat Q, this heat could be used to raise the temperature of water. The relation between Q and the change ΔT in the temperature of the water is $Q = cm\Delta T$ (Equation 12.4), where c is the specific heat capacity of water and m is its mass. Solving this expression for m and substituting Equation (1) for $W_{\text{ground-cloud}}$, we have

$$m = \frac{Q}{c\Delta T} = \frac{W_{\text{ground-cloud}}}{c\Delta T} = \frac{-q\left(V_{\text{cloud}} - V_{\text{ground}}\right)}{c\Delta T} \qquad (3)$$

SOLUTION

a. The work done on the charge as it moves from the ground to the cloud is $W_{\text{ground-cloud}} = -q(V_{\text{cloud}} - V_{\text{ground}})$. Setting $q = -25$ C (the charge is negative) and $V_{\text{cloud}} - V_{\text{ground}} = 1.2 \times 10^9$ V, we find that the work is

$$W_{\text{ground-cloud}} = -q\left(V_{\text{cloud}} - V_{\text{ground}}\right) = -(-25\text{ C})\left(1.2\times10^{9}\text{ V}\right) = \boxed{3.0\times10^{10}\text{ J}}$$

b. From Equation (2) we have for the speed of the automobile that

$$v_{\text{f}} = \sqrt{\frac{-2q\left(V_{\text{cloud}} - V_{\text{ground}}\right)}{m}} = \sqrt{\frac{-2(-25\text{ C})\left(1.2\times10^{9}\text{ V}\right)}{1100\text{ kg}}} = \boxed{7.4\times10^{3}\text{ m/s}}$$

c. The mass of water that can be heated is given by $m = \frac{-q\left(V_{\text{cloud}} - V_{\text{ground}}\right)}{c\Delta T}$ [see Equation (3)]. Setting $\Delta T = 100$ C° and using $c = 4186$ J/(kg·C°) from Table 12.2, we find that the mass of water is

$$m = \frac{-q\left(V_{\text{cloud}} - V_{\text{ground}}\right)}{c\Delta T} = \frac{-(-25\text{ C})\left(1.2\times10^{9}\text{ V}\right)}{\left[4186\text{ J/}\left(\text{kg}\cdot\text{C°}\right)\right](100\text{ C°})} = \boxed{7.2\times10^{4}\text{ kg}}$$
