Homework-12

Chapter 26

26. ***REASONING*** The light ray traveling in the oil can only penetrate into the water if it does not undergo total internal reflection at the boundary between the oil and the water. Total internal reflection will occur if the angle of incidence $\theta = 71.4°$ is greater than the critical angle θ_c for these two media. The critical angle is found from

$$\sin\theta_c = \frac{n_2}{n_1} \qquad (26.4)$$

where $n_2 = 1.333$ is the index of refraction of water (see Table 26.1), and $n_1 = 1.47$ is the index of refraction of the oil.

SOLUTION Solving Equation 26.4 for θ_c, we obtain

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.333}{1.47}\right) = \boxed{65.1°}$$

Comparing this result to $\theta = 71.4°$, we see that the angle of incidence is greater than the critical angle ($\theta > \theta_c$). Therefore, $\boxed{\text{the ray of light will not enter the water}}$; it will instead undergo total internal reflection within the oil.

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31. ***REASONING AND SOLUTION***

a. Using Equation 26.4 and the refractive index for crown glass given in Table 26.1, we find that the critical angle for a crown glass-air interface is

$$\theta_c = \sin^{-1}\left(\frac{1.00}{1.523}\right) = 41.0°$$

The light will be totally reflected at point A since the incident angle of 60.0° is greater than θ_c. The incident angle at point B, however, is 30.0° and smaller than θ_c. Thus, the light will exit first at $\boxed{\text{point B}}$.

b. The critical angle for a crown glass-water interface is

$$\theta_c = \sin^{-1}\left(\frac{1.333}{1.523}\right) = 61.1°$$

The incident angle at point A is less than this, so the light will first exit at $\boxed{\text{point A}}$.

36. ***REASONING*** Using the value given for the critical angle in Equation 26.4 (sin $\theta_c = n_2/n_1$), we can obtain the ratio of the refractive indices. Then, using this ratio in Equation 26.5 (Brewster's law), we can obtain Brewster's angle θ_B.

SOLUTION From Equation 26.4, with $n_2 = n_{air} = 1$ and $n_2 = n_{liquid}$, we have

$$\sin\theta_c = \sin 39° = \frac{1}{n_{liquid}} \tag{1}$$

According to Brewster's law,

$$\tan\theta_B = \frac{n_2}{n_1} = \frac{1}{n_{liquid}} \tag{2}$$

Substituting Equation (2) into Equation (1), we find

$$\tan\theta_B = \frac{1}{n_{liquid}} = \sin 39° = 0.63 \qquad \text{or} \qquad \theta_B = \tan^{-1}(0.63) = \boxed{32°}$$

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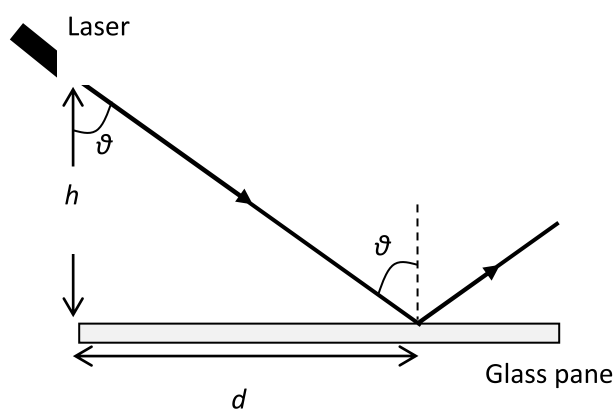
38. ***REASONING*** Given the height h of the laser above the glass pane, the distance d between the edge of the pane and the point where the laser reflects depends upon the angle θ between the incident ray and the vertical (see the drawing). The height h is adjacent to the angle θ, and the distance d is opposite, so from $\tan\theta = \frac{d}{h}$ (Equation 1.3) we have that



$$d = h\tan\theta \tag{1}$$

The glass pane is horizontal, so the normal to the surface is vertical, and we see that θ is also the angle of incidence when the incident ray strikes the glass pane. The reflected beam is 100% polarized, so we conclude that the angle θ of incidence is equal to Brewster's angle θ_B for the glass-air interface: $\theta = \theta_B$. Brewster's angle is given by $\tan\theta_B = \frac{n_2}{n_1}$ (Equation 26.5), where $n_2 = 1.523$ and $n_1 = 1.000$ are the indices of refraction for crown glass and air, respectively (see Table 26.1). Therefore,

$$\tan\theta = \tan\theta_{\text{B}} = \frac{n_2}{n_1} \qquad (2)$$

SOLUTION Substituting Equation (2) into Equation (1), we obtain

$$d = h\tan\theta = h\left(\frac{n_2}{n_1}\right) = (0.476\text{ m})\left(\frac{1.523}{1.000}\right) = \boxed{0.725\text{ m}}$$

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43. ***REASONING*** The angle of each refracted ray in the crown glass can be obtained from Snell's law (Equation 26.2) as $n_{\text{diamond}}\sin\theta_1 = n_{\text{crown glass}}\sin\theta_2$, where θ_1 is the angle of incidence and θ_2 is the angle of refraction.

SOLUTION The angles of refraction for the red and blue rays are:

Blue ray $$\theta_2 = \sin^{-1}\left(\frac{n_{\text{diamond}}\sin\theta_1}{n_{\text{crown glass}}}\right) = \sin^{-1}\left[\frac{(2.444)\sin 35.00^\circ}{1.531}\right] = 66.29^\circ$$

Red ray $$\theta_2 = \sin^{-1}\left(\frac{n_{\text{diamond}}\sin\theta_1}{n_{\text{crown glass}}}\right) = \sin^{-1}\left[\frac{2.410\sin 35.00^\circ}{1.520}\right] = 65.43^\circ$$

The angle between the blue and red rays is

$$\theta_{\text{blue}} - \theta_{\text{red}} = 66.29^\circ - 65.43^\circ = \boxed{0.86^\circ}$$

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50. ***REASONING***

a. Given the focal length ($f = -0.300$ m) of the lens and the image distance d_{i}, we will employ the thin-lens equation $\frac{1}{d_{\text{o}}} + \frac{1}{d_{\text{i}}} = \frac{1}{f}$ (Equation 26.6) to determine how far from the van the person is actually standing, which is the object distance d_{o}. The image and the person are both behind the van, so the image is virtual, and the image distance is negative: $d_{\text{i}} = -0.240$ m.

b. Once we have determined the object distance d_o, we will use the magnification equation $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$ (Equation 26.7) to calculate the true height h_o of the person from the height $h_i = 0.34$ m of the image.

SOLUTION
a. Solving Equation 26.6 for d_o, we obtain

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} \qquad \text{or} \qquad d_o = \frac{1}{\frac{1}{f} - \frac{1}{d_i}} = \frac{1}{\frac{1}{-0.300\text{ m}} - \frac{1}{(-0.240\text{ m})}} = \boxed{1.2\text{ m}}$$

b. Taking the reciprocal of both sides of $\frac{h_i}{h_o} = -\frac{d_i}{d_o}$ (Equation 26.7) and solving for h_o yields

$$\frac{h_o}{h_i} = -\frac{d_o}{d_i} \qquad \text{or} \qquad h_o = -h_i\left(\frac{d_o}{d_i}\right) = -(0.34\text{ m})\left(\frac{1.2\text{ m}}{-0.240\text{ m}}\right) = \boxed{1.7\text{ m}}$$

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52. ***REASONING*** The height of the mountain's image is given by the magnification equation as $h_i = -h_o d_i/d_o$. To use this expression, however, we will need to know the image distance d_i, which can be determined using the thin-lens equation. Knowing the image distance, we can apply the expression for the image height directly to calculate the desired ratio.

SOLUTION According to the thin-lens equation, we have

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \qquad (1)$$

For both pictures, the object distance d_o is very large compared to the focal length f. Therefore, $1/d_o$ is negligible compared to $1/f$, and the thin-lens equation indicates that $d_i = f$. As a result, the magnification equation indicates that the image height is given by

$$h_i = -\frac{h_o d_i}{d_o} = -\frac{h_o f}{d_o} \qquad (2)$$

Applying Equation (2) for the two pictures and noting that in each case the object height h_o and the focal length f are the same, we find

$$\frac{(h_i)_{5\text{ km}}}{(h_i)_{14\text{ km}}} = \frac{\left(-\frac{h_o f}{d_o}\right)_{5\text{ km}}}{\left(-\frac{h_o f}{d_o}\right)_{14\text{ km}}} = \frac{(d_o)_{14\text{ km}}}{(d_o)_{5\text{ km}}} = \frac{14\text{ km}}{5.0\text{ km}} = \boxed{2.8}$$