Physics 201 Make-Up Lab: Standing Waves on a String

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Fall, 2008

Theoretical Discussion

Figure 1 shows a wave traveling in the positive x-direction. The wave is a function of both x and t:

$$y = f(x, t)$$

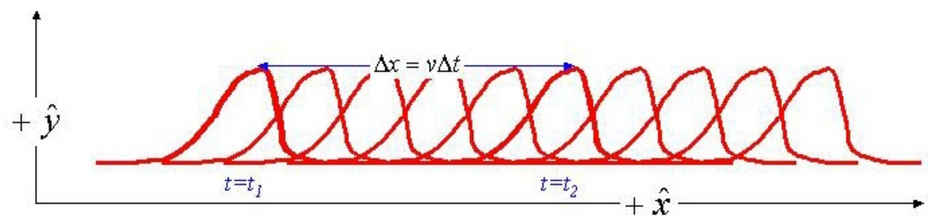

FIG. 1: A periodic wave traveling in the positive x-direction

The value of y at any two points x_2 and x_1, at the times t_2 and t_1, are exactly the same, so long as

$$\Delta x = x_2 - x_1 = v(t_2 - t_1) = v\Delta t$$

The velocity v is known as the *phase velocity* of the wave. It is apparent, therefore, that

$$f(x + v\Delta t, t + \Delta t) = f(x, t)$$

which implies that the functional form of y is

$$y = f(x - vt) \tag{1}$$

The meaning of this equation is that the size of the wave disturbance—the value of y—is the same for any two values of (x_1, t_1) and (x_2, t_2) so long as

$$f(x_2 - vt_2) = f(x_1 - vt_1)$$

Equation 1 is the most general form for the equation of a traveling wave. A particularly important class of traveling waves are *periodic* waves. These are traveling waves which are generated by a regular, periodic source, so that at any given point in space, after some period of time $\Delta t = T$, the wave will recur; similarly, at any given point in time, the wave pattern will be recreated at spatial intervals of $\Delta x = \lambda$. The time interval T is called the *period* of the wave, and the space interval λ is called its *wavelength.* A general periodic traveling wave can be constructed by adding together an infinite sum of sine and cosine waves, each term of which obeys equation 1. Any given term of the wave will have the form of a *simple harmonic wave*, namely

$$y = y_m \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} + \phi\right) \tag{2}$$

where y_m is the amplitude, or maximum height of the wave, and ϕ is called the phase angle. It gives the value of the wave when $x = t = 0$. We shall henceforth take $\phi = 0$. The reciprocal of the period, $f = \frac{1}{T}$, is called the frequency of the wave. One can re-write equation 2 in the form

$$y = y_m \sin\left(\frac{2\pi}{\lambda}[x - \lambda f t]\right) \tag{3}$$

Comparison of equation 3 with equation 1 implies that the phase velocity is given by

$$v = \lambda f = \frac{\lambda}{T} \tag{4}$$

You can verify, using your knowledge that $\sin(\theta + 2\pi) = \sin\theta$, that $y(x + \lambda, t) = y(x, t)$ for any time t, and $y(x, t + T) = y(x, t)$ for any position x.

The phase velocity v of a simple harmonic wave traveling along a vibrating string is related to the tension in the string τ according to

$$v = \sqrt{\frac{\tau}{\mu}} \tag{5}$$

where μ is the mass per unit length of the string.

In the general case, when an oscillating source is driving the string at one end, and the string is fixed at the other end, the various harmonic components of the traveling wave will be reflected back from the fixed end of the string with a variety of phase angles ϕ. Unless the difference in phase between the waves traveling in the $+x$-direction and the reflected waves traveling in the $-x$-direction is equal to an integral multiple of 2π, they will destructively interfere—cancel out, in other words. If the waves going in either direction are *in phase*, meaning that their arguments differ by an integer multiple of 2π, then they will constructively interefere. In that case, the amplitude of the resulting wave will be maximal. We call such a situation a *standing wave*.

For constructive interference to occur, there must exist a specific relationship between the phase of the waves at the two ends of the string; namely, the length L of the string must be equal to an integral number of half-wavelengths, as shown in figure 2.

$$L = n\frac{\lambda}{2} \quad n = 1, 2, 3, \ldots \tag{6}$$

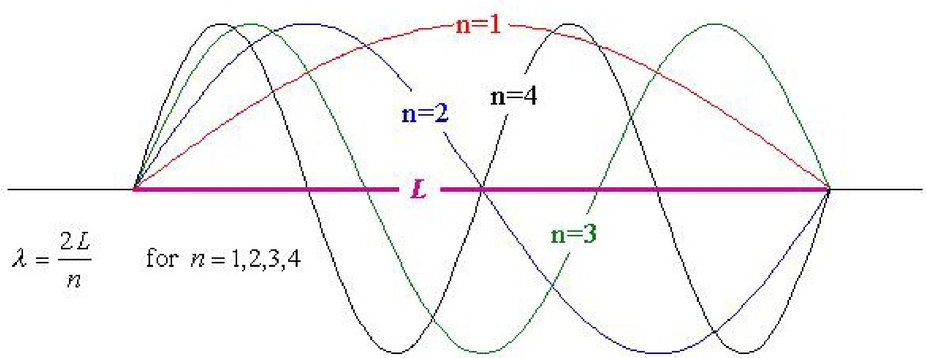

FIG. 2: Conditions for constructive interference on a string with fixed endpoints

The various allowed wavelengths $\lambda_n = \frac{2L}{n}$ are called the *modal wavelengths*. Since the frequency f is fixed by the frequency of the wave generator, each mode has a different velocity, which corresponds to a different string tension τ_n. Combining equations 4, 5, and 6, the relationship between the allowed modal wavelength and the requisite string tension is given by

$$\lambda_n = \frac{v_n}{f} = \frac{1}{f}\sqrt{\frac{\tau_n}{\mu}} \tag{7}$$

Equation 6 tells us that the mode number n is given by $n = \frac{2L}{\lambda_n}$. Substituting this result into equation 7, we obtain an expression for the square of the mode number:

$$n^2 = 4L^2 f^2 \frac{\mu}{\tau_n} \tag{8}$$

Procedure

In today's lab, you will find the allowed modes of a string vibrating at a fixed frequency by varying the tension required to create each mode. The tension is varied by adjusting the mass hung from a hanger attached to one end of the string. The tension in the string is therefore equal to $\tau = mg$. The experimental setup is shown in figure 3. You will find the mass m_n required to create the n^{th} standing wave pattern, where the wavelength of this mode is equal to $\lambda_n = \frac{2L}{n}$, for $n = 1, 2, 3$, and 4. The tension in the string for this mode is equal to

$$\tau_n = m_n g \tag{9}$$

Substituting the expression in equation 9 for the tension in equation 8, one obtains a relation between the square of the mode number and the mass m_n required to generate that mode:

$$n^2 = 4L^2 f^2 \frac{\mu}{g}\left(\frac{1}{m_n}\right) \tag{10}$$

You will then plot, by hand, the square of the mode numbers vs. the reciprocal of the experimentally determined masses m_n.

The slope s of the plot of n^2 vs. $\frac{1}{m_n}$ will therefore be equal to

$$s = 4L^2 f^2 \frac{\mu}{g} \tag{11}$$

Algebraically re-arranging equation 11, one obtains an expression for the oscillator frequency f:

$$f = \frac{1}{2L}\sqrt{\frac{sg}{\mu}} \tag{12}$$

Note: The mass per unit length of the string is $\mu = 2.62 \times 10^{-4}$ kg/m.

1. Measure the length of the string L.
2. For modes corresponding to $n = 1, 2, 3$, and 4, adjust the mass on the hanger until you achieve the resonance condition (large amplitude modulation) for the modal wavelength $\lambda = \frac{2L}{n}$. Record the mass m_n for this mode. Be sure to include the mass of the hanger.

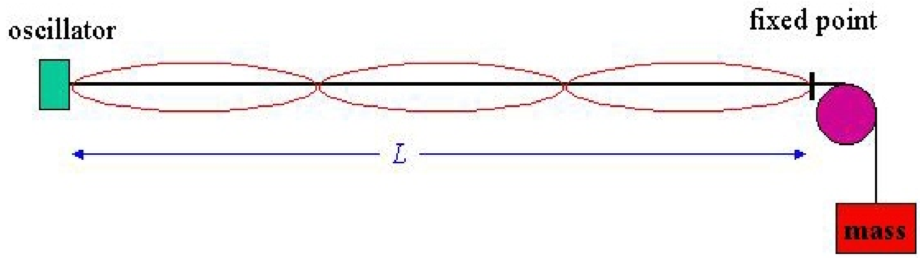

FIG. 3: Experimental setup for measurement of the frequency of an oscillating string

3. Plot n^2 vs. $\frac{1}{m_n}$ and determine the slope s of this line.
4. Determine the frequency by substituting the experimental slope into equation 12.

Report

1. Tabulate all of your measured data; L, n and m_n.
2. Plot n^2 vs. $\frac{1}{m_n}$ and determine the slope s of this line.
3. Calculate and report the frequency f.