Physics 201 Lab 5: Rotational Motion

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Fall, 2008

Theoretical Discussion

An object moving in a circular orbit[1] at constant speed is said to be executing *uniform circular motion.* The kinematic variables that describe an object in uniform circular motion are the radius R of the orbit and the frequency f (or ν) of the orbit. The frequency refers to the number of revolutions per unit time that the object makes about the axis of rotation. It is closely related to the angular velocity ω, which describes the number of radian degrees $\Delta\theta$ per unit time Δt that the rotating object passes through while following its trajectory. Since there are 2π radians in a complete circle, the angular velocity is

$$\omega = \frac{\Delta\theta}{\Delta t} = \left(2\pi \frac{\text{radians}}{\text{revolution}}\right)\left(f\,\frac{\text{revolution}}{\text{second}}\right) = 2\pi f \text{ rad/s}$$

The arclength Δs subtended by the path of an object moving in a circular trajectory of radius R as it undergoes an angular displacement of $\Delta\theta$ is numerically equal to

$$\Delta s = R\Delta\theta$$

so long as $\Delta\theta$ is given in radians. It follows that the magnitude of the tangential velocity v, which is the instantaneous rate of change of the object's displacement as it follows its circular trajectory, is given by

$$v = \frac{\Delta s}{\Delta t} = R\frac{\Delta\theta}{\Delta t} = R\omega$$

The direction of the tangential velocity vector is, not surprisingly, tangential to the circular orbit. The geometry describing these rotational variables is shown in figure 1.

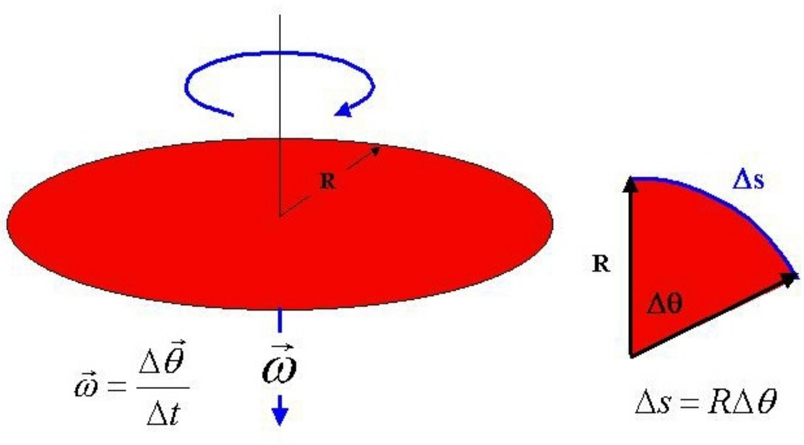

FIG. 1: Geometry describing rotational kinematic variables

A dynamically isolated object—one that does not have any external forces acting upon it—will travel at a constant speed in a straight line forever. The *only* way that an object can execute uniform circular motion, is for a force to act on it in such a way that its speed never changes, but the direction of its velocity does. Consider an object of mass m which has a velocity of $\vec{v}(t_0)$ at time t_0, shown in figure 2. At some later time

$t_1 = t_0 + \Delta t$, the velocity is $\vec{v}(t_1)$. The magnitudes of $\vec{v}(t_0)$ and $\vec{v}(t_1)$ are the same, but their directions are different. By definition, the average force $\vec{F}_{avg}$ that turns $\vec{v}(t_0)$ into $\vec{v}(t_1)$ is given by

$$\vec{F}_{avg} = m\frac{\Delta \vec{v}}{\Delta t} = m\frac{\vec{v}(t_1) - \vec{v}(t_0)}{t_1 - t_0}$$

so that the three vectors $\vec{v}(t_0)$, $\vec{v}(t_1)$ and $\frac{(t_1 - t_0)}{m}\vec{F}_{avg}$ form the triangle shown in figure 2. As the time interval Δt becomes smaller and smaller, as shown in figures 3A and 3B, the instantaneous force is seen to be directed towards the center of the circle. The magnitude of the instantaneous force is equal to

$$F_{inst} \ = \ m\frac{v^2}{R} = m\omega^2 R = m(2\pi f)^2 R \qquad (1)$$

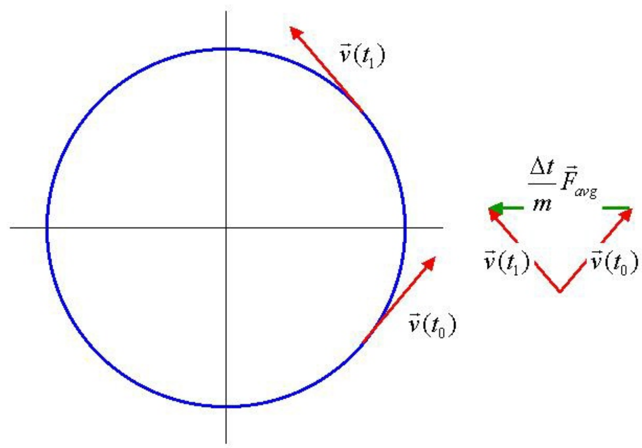

FIG. 2: Change in tangential velocity over a finite time interval

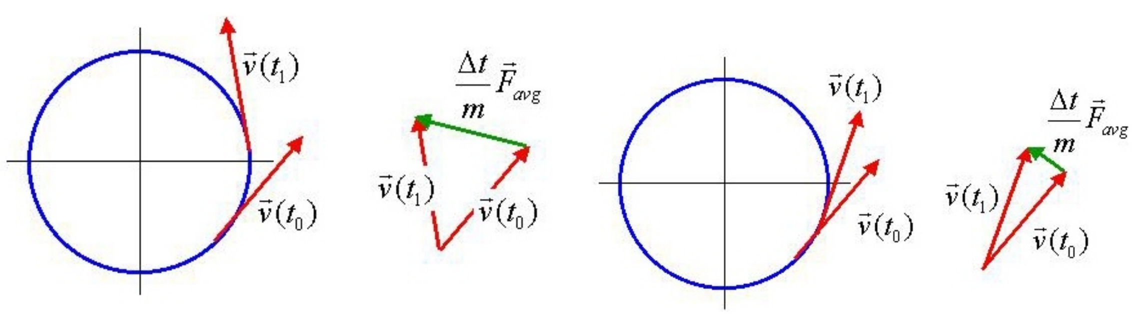

Figure 3A　　　　　Figure 3B

FIG. 3: Direction of centripetal force in the limit as $\Delta t \rightarrow 0$

Procedure

In today's lab, you will determinine (among other things) the magnitude of the instantaneous force required to keep a massive object in uniform circular motion. The experimental set-up is shown in figure 4. You will spin the rotor arm from which the mass is suspended at a constant rate of angular rotation so that the radius of the circular orbit of the mass is kept constant. You can ensure this by keeping the pointed end of the mass vertically aligned with the pointer. It is important that the mass is hanging straight up and down while you are making your measurement, because in this case, the string from which the mass is suspended exerts no component of force along the direction of the radius of the orbit; the force keeping the mass in uniform circular motion is therefore supplied completely by the spring.

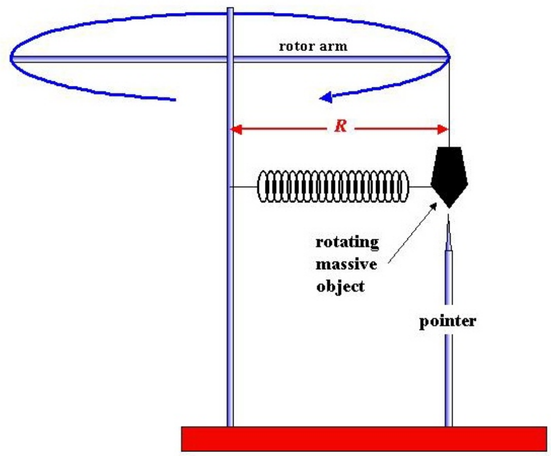

FIG. 4: Experimental setup for the measurement of centripetal force

The radius of the orbit R is equal to the distance from the axis of rotation to the pointer when the mass is disconnected from the spring and therefore, free to hang straight down. You should align the pointer so that it is vertically aligned with the pointed end of the mass when the spring is disconnected. After re-connecting the spring, spin the rotor arm, at as constant a rate as possible, so that the mass point is once again vertically aligned with the pointer. When this condition is achieved, you should time (using a stopwatch) how long it takes for the mass to execute 20 rotations. It would be ideal to have two timers make this measurement and then average the two times. From this average time, along with your measurement of the orbital radius R, you can calculate the frequency f, the angular frequency ω and the tangential speed v according to

$$\langle t \rangle = \frac{t_1 + t_2}{2} \quad (2)$$

$$f = \frac{20}{\langle t \rangle} \text{ rev/s} \quad (3)$$

$$\omega = 2\pi f \text{ rad/s} \quad (4)$$

$$v = \omega R \text{ m/s} \quad (5)$$

Using these results, you can calculate a predicted value for the instantaneous force F_{pred} acting on the mass:

$$F_{pred} = m\omega^2 R \quad (6)$$

where m is the mass of the rotating object. (Measure this mass without the spring attached, but with the string from which it is suspended from the rotor arm). In order to check your prediction, you must make an independent measurement of the force exerted by the spring when the mass is vertically suspended over the pointer.

The magnitude of the force F_{spring} exerted by a spring when it is displaced a distance x from its equilibrium position is equal to

$$F_{spring} = k_s x$$

where k_s is a parameter called the *spring constant* that, in general, depends on the particular spring you are using. The dimensions of k_s are N/m in SI units. It is not necessary, however, to determine k_s in order to measure the force exerted by the spring during the course of the experiment. All that is necessary is to determine the magnitude of the force W is required to extend the spring until the mass is vertically suspended over the pointer when the rotor arm is not turning. As shown in figure 5, if we attach a mass hanger to the rotating mass and drape the hanger over a pulley, then suspend some weights from the mass hanger until the rotating object is vertically aligned with the pointer, the force W produced by gravitational force acting on the mass M suspended from the pulley (including the mass of the hanger), is equal and opposite to the force exerted by the spring when the object is rotating.

$$F_{exp} = F_{spring} = W = Mg \quad (7)$$

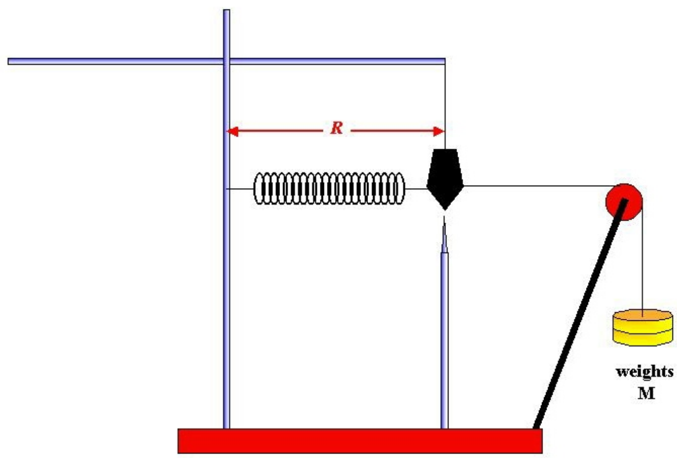

FIG. 5: Method for determining centripetal force by imposing static equilibrium with gravitational forces

Report

1. Tabulate all of your measured data;
   - The mass m of the rotating object.
   - The radius R of the object's trajectory.
   - The time intervals t_1, and t_2 for the object to execute 20 rotations, as measured by your two timers, as well as the average time interval $\langle t \rangle$.
   - The mass M of the weight (including the hanger) required to counterbalance the spring force and position the object over the pointer.
2. Tabulate all of your calculated results;
   - The mean period $\tau = \frac{\langle t \rangle}{20}$.
   - The frequency f.
   - The angular velocity ω.

- The predicted force F_{pred}.
- The measured force F_{exp}.

3. Calculate and report the fractional discrepancy between F_{pred} and F_{exp}.

Hint: *It would be a good idea to convert all of your measurements to standard SI units (kg, m, s) before making your calculations.*

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[1] Circular motion can be defined as continuous motion in a plane that maintains a constant distance R from some fixed point P in the plane.