Physics 201 Lab 11: The pendulum and simple harmonic motion

Dr. Timothy C. Black

Fall, 2008

Theoretical Discussion

The first truly accurate time-keeping devices were pendulum clocks. Although they are not considered accurate by today's standards, in their time, they effected a revolution in both economics and scientific investigation. Among their many virtues was that, unlike hourglasses or water clocks, they did not require human intervention to turn the glass over or fill the water basin. They were more or less automatic. In today's lab, we will measure the period of oscillation of a couple of different pendula and compare these measurements with theoretical calculations.

Figure 1 depicts the experimental setup for the measurement of the period of a pendulum. As can be seen from the figure, the restoring force acts in a direction perpendicular to the instantaneous line of the pendulum supporting cord, which we are taking to be massless. On a downstroke, the restoring force is parallel to the velocity, which is in the direction of decreasing angle θ, whereas on an upstroke, the restoring force is anti-parallel to the instantaneous velocity, which is in the direction of increasing angle θ. Since the arclength s subtended by an angle θ on a circle of radius l is given by $s = l\theta$[1], the tangential velocity of the pendulum bob is

$$v = \frac{ds}{dt} = -l\frac{d\theta}{dt}$$

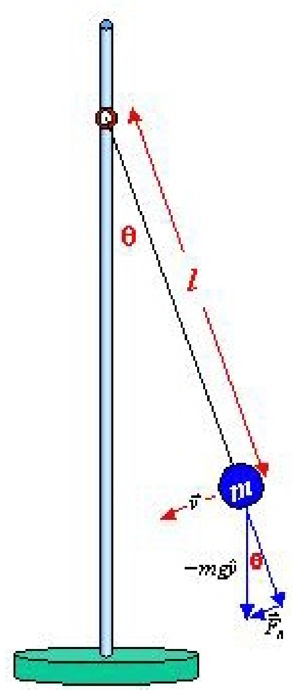

FIG. 1: Experimental setup for the measurement of the period of a pendulum

According to Newton's second law, the restoring force is related to the acceleration by

$$F_R = ma = m\frac{dv}{dt} = -ml\frac{d^2\theta}{dt^2} \tag{1}$$

Since the restoring force is equal to the tangential component of the gravitational mass of the pendulum bob,

$$F_R = mg \sin\theta \tag{2}$$

Equation the expressions for the restoring forces from equations 1 and 2 gives the equation of motion for the pendulum:

$$-ml\frac{d^2\theta}{dt^2} = mg \sin\theta \tag{3}$$

As it stands, this equation is not readily soluble. However, if we perform a Taylor series expansion of the sine function about the point $\theta_0 = 0$, we obtain the series

$$\sin\theta \approx \theta - \frac{1}{6}\theta^3 + \cdots$$

As long as the angle θ is kept small enough so that we need retain only the first term in the Taylor series expansion, then $\sin\theta \approx \theta$, and equation 3 becomes

$$-ml\frac{d^2\theta}{dt^2} \approx mg\theta \tag{4}$$

which we can readily solve. Cancelling the masses, and re-arranging the terms, we obtain

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \tag{5}$$

whose solution is the periodic function

$$\theta(t) = \theta_0 \cos(\omega t) \tag{6}$$

where $\omega = \frac{2\pi}{\tau}$ and τ is the period of oscillation. Differentiating equation 6 twice, one obtains

$$\frac{d^2\theta}{dt^2} = -\theta_0\omega^2 \cos(\omega t) \tag{7}$$

and comparing equation 7 to equation 5 allows us to identify the angular frequency as

$$\omega = \sqrt{\frac{g}{l}} \tag{8}$$

so that the period is

$$\tau = 2\pi\sqrt{\frac{l}{g}} \tag{9}$$

One facet of this equation that is very interesting is that the period is independent of the mass of the pendulum bob. We are going to verify this. In fact, it depends solely on the length of the pendulum l. We are going to verify this as well.

Procedure

We are going to measure the period of two pairs of pendula. The pendula in pair one will have a length of l_1, but two different pendulum bob masses m_1 and m_2. The second pair will have a length of $l_2 \neq l_1$, but will use the same bob masses m_1 and m_2. You will therefore be measuring the (average) periods of four different pendula, $\langle \tau_{11} \rangle$ and $\langle \tau_{12} \rangle$, corresponding to the first pair of pendula; and $\langle \tau_{21} \rangle$ and $\langle \tau_{22} \rangle$, corresponding to the second pair.

You can measure the average period of each pendulum by timing 20 complete oscillations with a stopwatch, and then dividing by this interval by 20. Thus the experimental average period for the ij^{th} pendulum is

$$\tau_{ij}^{\text{exp}} = \frac{\Delta t_{ij}}{20} \tag{10}$$

where Δt_{ij} is the time it takes for the pendulum to execute 20 oscillations. The procedure can be summarized as follows:

1. Carefully measure the length l of the pendulum. The length should run from the attachment point on the pendulum support to the approximate center of mass of the pendulum bob.
2. For each of the four pendula, measure the time for 20 oscillations, Δt_{ij}.
3. Use these measurements, and equation 10 to calculate the experimental values for the average periods $\langle \tau_{ij}^{\text{exp}} \rangle$.
4. For each of the four pendula, calculate the theoretical values for the ij^{th} period, τ_{ij}^{theo}, using equation 9. Obviously, $\tau_{i1}^{\text{theo}} = \tau_{i2}^{\text{theo}}$.
5. Numerically compare $\langle \tau_{ij}^{\text{exp}} \rangle$ to τ_{ij}^{theo} for each of the four pendula by calculating the fractional discrepancy between them.
6. Neatly and intelligently tabulate your data and results.

---

[1] so long as the angle θ is expressed in radians