Physics 201 Lab 6: Decomposition of Contact Interactions and the inclined plane

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Theoretical Discussion

Here's a couple of questions for you—If the two cars in figure 1 are identical Saturns, and they race down their respective frictionless tracks, which one will be going fastest at the bottom of the track, or will they be going the same speed? Which one will reach the bottom first?

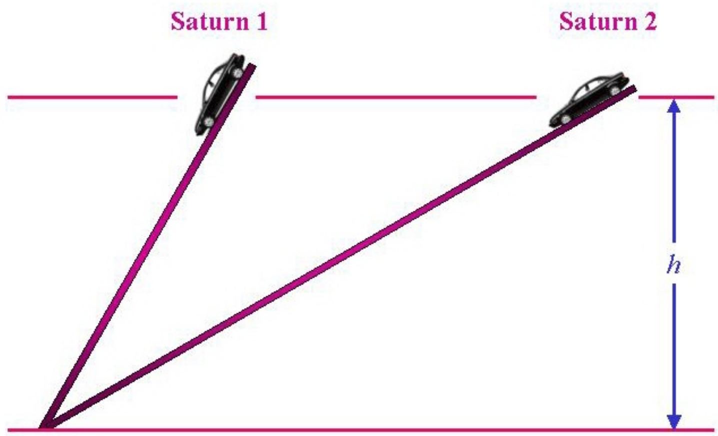

FIG. 1: Kinematics of two identical cars on inclined planes of equal height

If you answered that they would have the same speed at the bottom, you were absolutely right. The conservation of energy requires that if their initial kinetic energies are the same, and the change in their potential energies are the same, then their final kinetic energies are the same; if they have the same mass, then their final speeds are the same. It is pretty clear, however, that Saturn 1 is going to get to the bottom first. Is there a contradiction? In fact, there is no contradiction. Although they both experience the same *change* in their speeds, the *rates* at which their speeds change are very different. Saturn 1 goes from its initial to its final speed a lot faster than Saturn 2. However, since Saturn 2's track is longer, it has a longer time period over which to reach its final speed.

The time rate of change of velocity is called *acceleration*. The time rate of change of momentum is called *force*. If the mass of an object is constant, then the force is simply the mass times the acceleration. Since Saturn 1's acceleration is greater than Saturn 2's, it must have a greater net force acting on it. Yet the only external force in the system is gravity, and it is the same for both cars. How is it that the same external force can give rise to different net forces? The answer to this question can be found by considering the nature of contact interactions.

The situation is described graphically in figure 2A. The gravitational force acting on an object on an inclined plane can be broken into two components; one parallel and one perpendicular to the plane of the incline. The incline exerts a force, called the *normal force* that is equal and opposite to the perpendicular component of gravity, and therefore cancels it. But in the absence of friction, the inclined plane cannot exert a reaction force to compensate the parallel component of gravity. Therefore, the net force on the object is equal to the uncompensated parallel component.

$$\vec{F}_{\text{net}} = \vec{F}_G + \vec{F}_N = \vec{F}_{\parallel} + \underbrace{\vec{F}_{\perp} + \vec{F}_N}_{0} = \vec{F}_{\parallel}$$

.
If the $\hat{x}$-axis is chosen along the parallel direction, as in figure 2A, the net gravitational force on an object of mass m (near the surface of the Earth), lying on an inclined plane, is

$$\vec{F}_{\text{net}} = \vec{F}_{\|} = -F_G \sin\theta \hat{x} = -mg \sin\theta \hat{x}$$

.
where θ is the angle the incline makes with the horizontal. The geometry of the inclined plane is described in figure 2B. The fact the the angle θ between the direction of the gravitational force and the perpendicular component of the normal force is equal to the angle that the incline makes with the horizontal should be clear from the figure; the x-y coordinate system of the inclined plane is just the usual horizontal-vertical coordinate system, rotated through an angle θ.

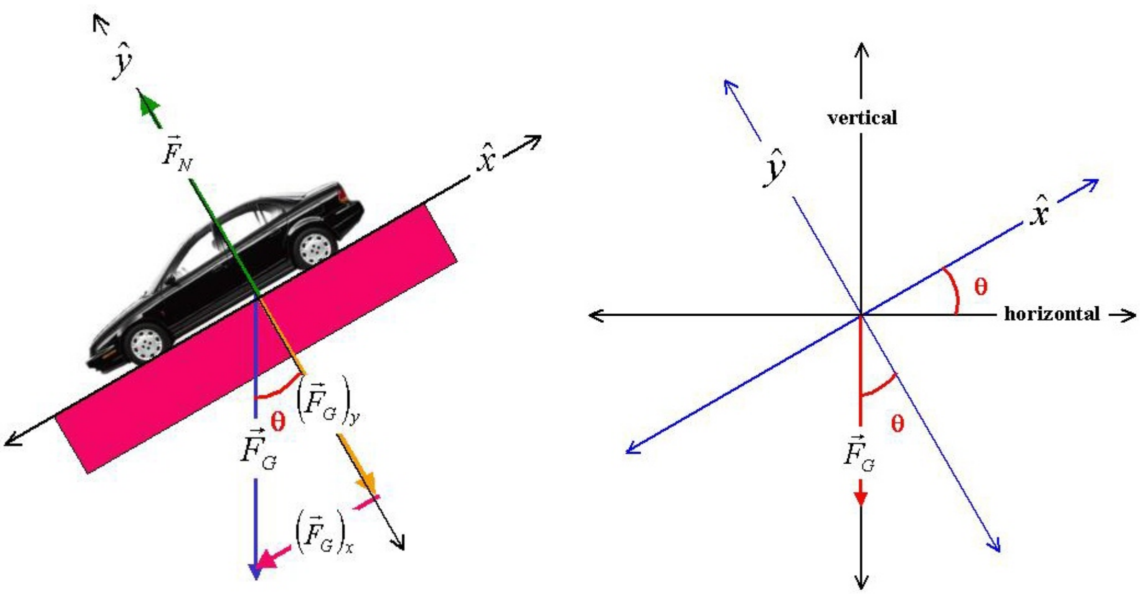

Figure 2A Figure 2B

FIG. 2: The decomposition of forces and geometry of the inclined plane

If no other forces act on the object, it will accelerate down the inclined plane, with an acceleration whose magnitude is $a = g \sin\theta$. As θ gets larger, the acceleration gets larger also, until at 90°—straight up and down—it is equal to g. In the ideal case of no friction, as soon as θ is greater than 0, the object will begin to accelerate down the plane. The fact that this does not usually happen in the real world—we do not all slide down to the lowest nearby point—is because of friction.

Friction acts at the interface between two surfaces to oppose the relative motion of those surfaces. It always acts parallel to the surface and it always opposes motion in any direction; whichever way you try to move an object, up or down, right or left, friction will oppose that motion. It has another peculiarity. Unlike forces like gravity, the frictional force is "variable" or "interactive" in that, for a body at rest, below a certain threshold it changes in magnitude so as to keep the net force equal to zero. For instance, if an object is stationary on an inclined plane, and you increase the tilt of the plane, thereby increasing the component of gravity parallel to the surface, the amount of friction opposing the downward motion increases also, so that the net force is zero. If you apply a light push *upwards* along the slope, the frictional force will act to oppose *that* also. However, the *static frictional force*, as the type of friction that opposes all motion is called, has an maximum strength, whose value depends on the nature of the surfaces that are in contact. Once this maximum is exceeded—by increasing the tilt beyond a certain angle, for instance—then the frictional force can no longer compensate enough to keep the object stationary. The frictional force “maxes out”. Friction, although it has been observed and described mathematically at an elementary level for several hundred years, is nonetheless very complex and is not, even today, completely understood at the most fundamental, microscopic level.

In the absence of friction, the only way to keep the object from accelerating down the incline is to pull on it from the opposite direction. A very convenient way to do this is with a pulley and some weights. Remember that a simple pulley does not change the magnitude of the string force, only its direction. Therefore, to keep an object—a box of mass m_b, for instance—stationary on an inclined plane, you can pull upward on it with the weight of a mass m_1 suspended from a pulley, as shown in figure 3, where the value of m_1 is given by

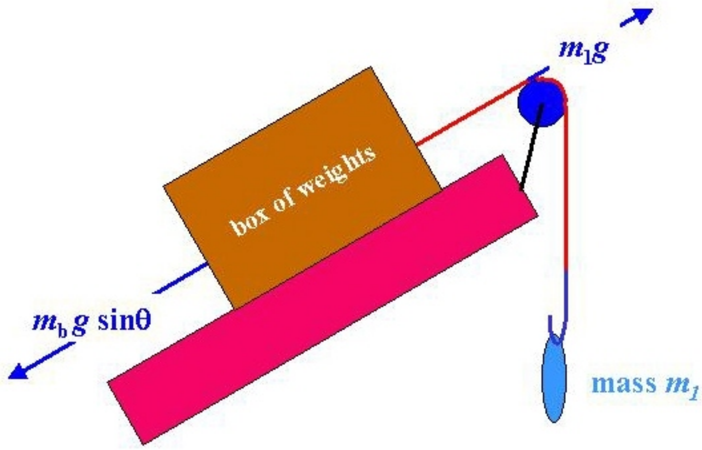

FIG. 3: Experimental setup for the inclined plane experiment

$$m_1 g = m_b g \sin\theta \tag{1}$$

If you set the inclined plane at a number of different angles, and measure the mass needed to balance the downward motion of the box, then plotted the mass $m_1(\theta)$ vs. $\sin\theta$, the slope of this line would be the mass of the box, m_b.

In the real world, however, there is friction. If you attempted this experiment, you would find that, for any given angle θ, there was a small range of masses m_1 for which you could achieve equilibrium. This is because friction opposes the motion of the box in either direction along the track. Suppose that the downward force of the box were slightly larger than the upward force of the weights. A small frictional force F_f would act in parallel with the weights, to keep the system in equilibrium. Taking the positive x-axis to be upwards along the track, one would have

$$m_1 g\hat{x} + F_f\hat{x} - m_b g \sin\theta\hat{x} = 0$$

If you decreased the mass m_1 to the point where the frictional force could no longer compensate, and the box began sliding down the track, then $m_1 = m_{1,\mathrm{min}}$ would be minimal, and the frictional force at that point, $F_{f,\mathrm{max}}$ would be at its maximum value. You would then have

$$m_{1,\mathrm{min}} g\hat{x} + F_{f,\mathrm{max}}\hat{x} - m_b g \sin\theta\hat{x} = 0 \tag{2}$$

On the other hand, if the upward force were slightly larger than the downward force, then the frictional force would be downward, and you would have

$$m_1 g\hat{x} - F_f\hat{x} - m_b g \sin\theta \hat{x} = 0$$

By increasing the amount of mass m_1 until the box began to slide *upward* along the track, you would again maximize the frictional force, but this time in the opposite direction.

$$m_{1,\text{max}} g\hat{x} - F_{f,\text{max}}\hat{x} - m_b g \sin\theta \hat{x} = 0 \tag{3}$$

Adding equation 2 to equation 3, then re-arranging and dividing by $2g$, gives you

$$\frac{m_{1,\text{max}} + m_{1,\text{min}}}{2} = m_b \sin\theta \tag{4}$$

Equation 4 provides the means to experimentally weigh the box with an inclined plane. By plotting the average of the minimum and maximum holding weights for a number of different angles θ, and then plotting these vs. $\sin\theta$, one can use the fact that the slope of this plot is equal to m_b to weigh the box. Note that in the limit that the frictional force goes to zero, $m_{1,\text{max}} = m_{1,\text{min}} = m_1$, and equation 4 reduces to equation 1. A summary of the experimental procedure is as follows:

1. Place approximately 500 g of loading weights inside the box.
2. For four different values of the incline angle; $\theta \approx 30°, 35°, 40°$ and $45°$; set up the box on the inclined plane as shown in figure 3. Use the angle marker on the side of the inclined plane to roughly set the angle. Then you can make a more precise measurement of the the angle using the digital level. Note that you will use the more precise measurement in your analysis. Do not attempt to adjust the incline of the plane to exactly match your target angle: this would take forever.
3. Estimate the amount of mass m_1 needed to balance the box and place this amount of mass on the hanger. Remember to include the mass of the hanger. You should assume that the string itself is massless.
4. At each angle, find the minimum and maximum values of the mass, $m_{1,\text{min}}$ and $m_{1,\text{max}}$ for which you can keep the box in equilibrium.
5. Plot $\frac{m_{1,\text{max}} + m_{1,\text{min}}}{2}$ vs. $\sin\theta$ and use the slope to find the mass of the box, m_b.
6. Weigh the box (with the loading weights still inside) on a mass balance.
7. Calculate the fractional discrepancy between the mass of the box determined from the slope, m_b, and the mass of the box as measured on the mass balance.
8. Neatly and intelligently tabulate your data and results.