Physics 201 Lab 4: Static equilibrium and superposition of forces

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Fall, 2008

Theoretical Discussion

Whenever a *net* force acts on an object, the object accelerates. Yet despite the fact that many of the objects around us in everyday life; our televisions, stereos, computers, sofas, refrigerators, etc.; are constantly subject to a large number of forces, they are not flying around in a state of perpetual acceleration. The reason that they are not is that usually, the *net* force on them is zero. A situation in which a number of non-zero forces combine to give a net force of zero is called *static equilibrium*. The manner in which forces combine is called **vector superposition**. The superposition principle for forces can be summarized as follows:

The net force that results from a number of individual forces acting on an object is the vector sum of the individual forces.

In mathematical terms, if N different forces act on an object, the net force on the object is equal to

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$$

Note that this is a vector equation, so that one must add the force vectors, and not merely the magnitudes of the forces. Therefore, the direction of the forces is very important. Both string forces and contact forces ultimately derive from electrostatic interactions, but the manner in which forces act through them is different. We can summerize these differences as follows:

- **Tension forces:** Tension forces, such as the forces mediated by a string, acts along the straight line defined by the direction of the string at the point where it is in contact with the object. Since a string or cord is not rigid, one can only pull, not push, with a string. Therefore the direction of the force is always away from the object. Tension force directions are shown in figure 1A.
- **Contact forces:** The force acting between two solid bodies in contact acts at every point of contact between them, and *only* at points of contact between them. The direction of the force is along the line connecting the two points of contact. For every point of contact, each body exerts an equal and opposite force on the other. The nature of contact forces are depicted in figure 1B.

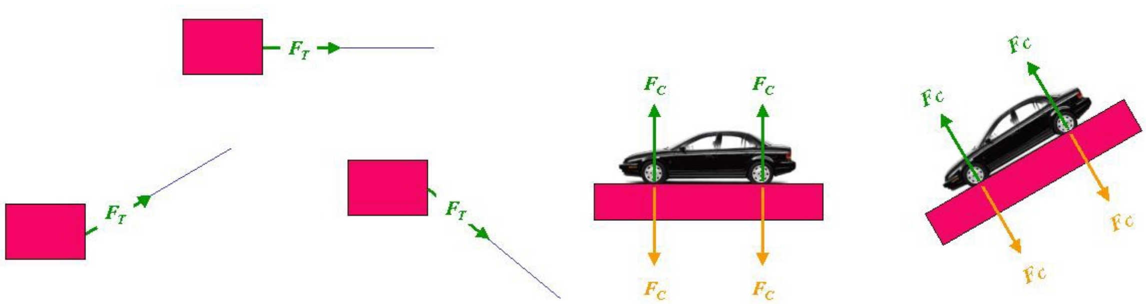

Figure 1A Figure 1B

FIG. 1: The directions of tension and contact forces

Since this experiment entails the use of pulleys, it is important to understand the manner in which pulleys mediate forces. A pulley acts only the change the direction of a tension force. A simple pulley does not change the magnitude of the force. The action of pulleys is described in figure 2.

If the system is in static equilibrium, then the net force is zero, so that

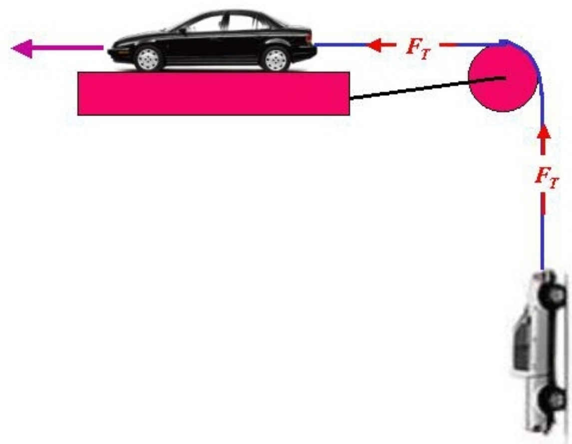

FIG. 2: Action of a simple pulley to change the direction of a tension force

$$\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N = 0$$

In two dimensions, the vector equation above can be written in terms of components to give the two equations

$$
\begin{aligned}
(\vec{F}_1)_x + (\vec{F}_2)_x + \cdots + (\vec{F}_N)_x &= 0 \\
(\vec{F}_1)_y + (\vec{F}_2)_y + \cdots + (\vec{F}_N)_y &= 0
\end{aligned}
$$

Both of these equations must be satisfied for the system to be in static equilibrium.

PROCEDURE

We are going to determine the mass of a test object, m_t, using the principle of superposition of forces and static equilibrium. The way we can do this is to use the force table to balance vector components of the gravitational forces due to 3 different masses; two reference masses—m_1 and m_2; and the test mass m_t. An overhead schematic diagram of the force table is shown in figure 3A. Figure 3B shows a side view of the force table.

If the masses are aligned as in figure 3A, then balancing the x and y components of the forces requires the following two equations to be satisfied.

$$
\begin{aligned}
m_1 g \cos\theta_1 + m_2 g \cos\theta_2 &= m_t g \qquad (1) \\
m_1 g \sin\theta_1 - m_2 g \sin\theta_2 &= 0 \qquad (2)
\end{aligned}
$$

You are free to make an arbitrary choice for θ_1. Once you do, then according to equation 2—which gives the condition for equilibrium for the y-component of force—θ_2 must be equal to

$$\theta_2 = \sin^{-1}\left(\frac{m_1}{m_2}\sin\theta_1\right)$$

Once you have determined θ_1 and θ_2, then substitution of these values into equation 1 yields the value for m_t that satisfies the condition for equilibrium for the x-component of force. The procedure can be summarized as follows:

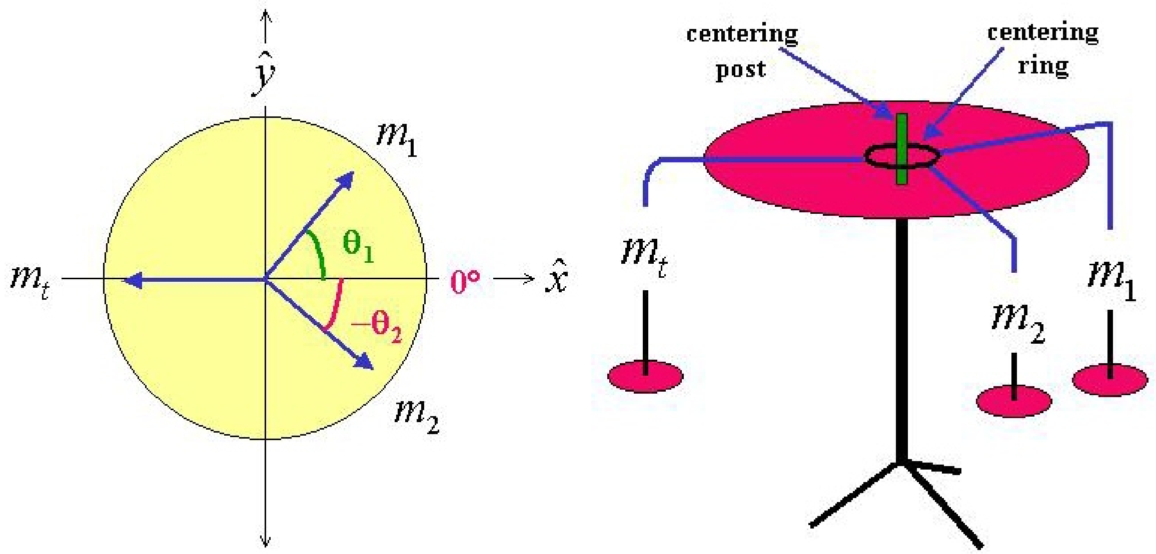

Figure 3A

Figure 3B

FIG. 3: Overhead and side views of the force table

1. Weigh hanger 1 on the mass balance; this mass is m_1. Weigh hanger 2 on the mass balance; this mass is m_2. The mass of the hangers is supposed to be 50 g, but naturally, you should not believe this and should weigh them yourself with the balance and record the measured values. The test mass should be attached to the third hanger.

2. Place the test mass at $\theta_B = 180°$. Adjust the angles of θ_1 and θ_2, so that the centering post is aligned in the middle of the centering ring. When this condition is achieved, the system is in static equilibrium, as shown in figure 3A. Record the values of θ_1 and θ_2.

3. Use your measured values of θ_1, θ_2, m_1 and m_2 to check that the y-component of the force equals zero.

4. Use the x-component equilibrium equation to calculate m_t.

5. Use the mass balance to measure m_t and see if it agrees with the value you determined by using the above procedure. You should numerically compare the values of m_t obtained using the two different methods by calculating the fractional discrepancy between them.

6. Neatly and intelligently tabulate your data and results.