Physics 201 Lab 7: The Conservation of Energy

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Theoretical Discussion

The relationship between potential energy and force: The change in potential energy ΔU of an object as it moves through a field of force is given by

$$\Delta U = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{F} \cdot \vec{ds} \tag{1}$$

Where $\vec{F}$ is the force and $\vec{ds}$ is the (vector) differential element of path in going from point $\vec{r_1}$ to point $\vec{r_2}$. In case the force is constant in both direction and magnitude over the entire path, equation 1 reduces to

$$\Delta U = -\vec{F} \cdot \left[\int_{\vec{r_1}}^{\vec{r_2}} \vec{ds} \right] = -\vec{F} \cdot \vec{r} \tag{2}$$

where $\vec{r} = \vec{r_2} - \vec{r_1}$ is the displacement vector from the beginning of the path to the end of the path. Since the scaler product of two vectors is equal to the product of the vector magnitudes and the cosine of the smallest angle θ between them, in case the force is constant in magnitude and direction everywhere along the path, the change in potential energy is given by

$$\Delta U = -\vec{F} \cdot \vec{r} = Fr\cos\theta \tag{3}$$

The conservation of energy: The "law" of conservation of energy states that in a closed system, the total energy of the system—which is equal to the sum of its potential and kinetic energy—is constant in time. It therefore follows that the sum of the change in potential energy ΔU and the change in kinetic energy ΔK, is equal to zero:

$$\begin{aligned} \Delta E &= \Delta U + \Delta K = 0 \\ &\Downarrow \ \Delta U = -\Delta K \end{aligned}$$

Applying this principle to the system described by equation 3 shows that the change in kinetic energy is equal to

$$\Delta K = Fr\cos\theta \tag{4}$$

The non-relativistic kinetic energy of an object of mass m, moving at a speed v, is given by

$$K = \frac{mv^2}{2} \tag{5}$$

so that an object which is initially at rest, having been displaced by the vector $\vec{r}$ through a constant force field $\vec{F}$, will have a final kinetic energy given by

$$K_{\text{fin}} = \frac{mv^2}{2} = Fr\cos\theta \tag{6}$$

Figure 1 depicts the relative directions of the gravitational force and the displacement vector for an object which is permitted to travel down an incline under the influence of gravity.

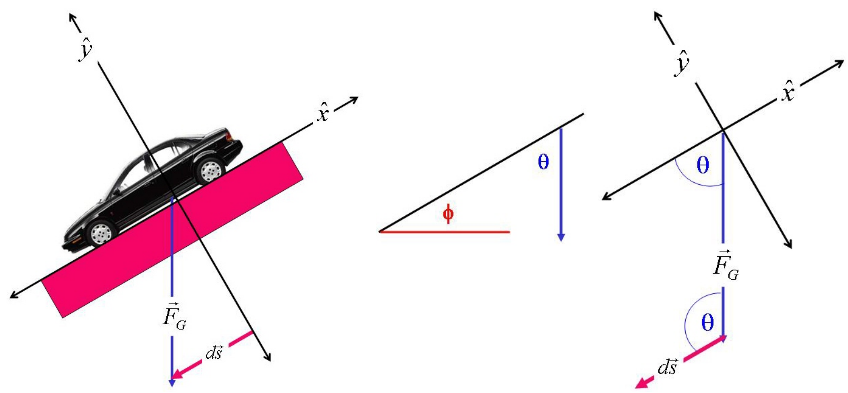

FIG. 1: Relative directions of the gravitational force $\vec{F_G}$ and the differential displacement vector $d\vec{s}$

Since the angle θ between the force and displacement vectors and the incline angle ϕ are complementary angles, it follows that

$$K_{fin} = Fr\cos\theta = Fr\sin\phi \tag{7}$$

PROCEDURE

The experimental set-up is shown in figure 2. You will allow a cart to glide down the air track through a fixed displacement r from its starting point to the photogate timer. You will measure both the final speed of the cart and the inclination angle, repeating the measurement for various inclination angles.

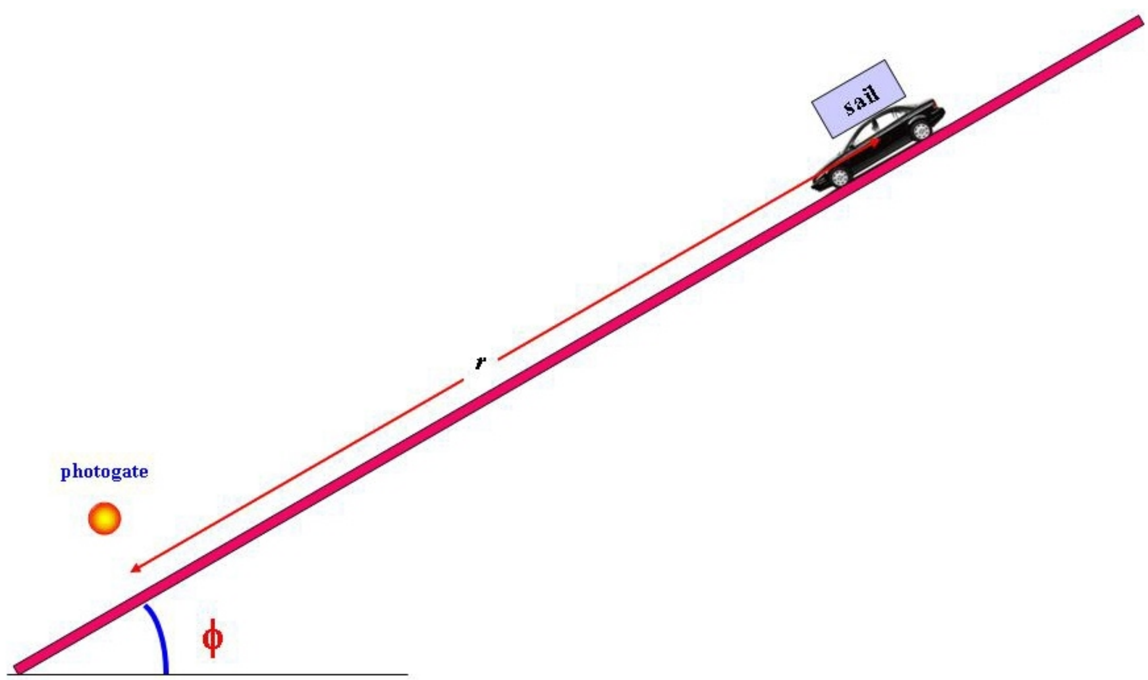

FIG. 2: Experimental set-up for measuring kinetic energy on an inclined plane.

The method of measuring the cart's speed is the same as usual: having measured the width w of the sail, we will use the photogate timers in *s1* mode to measure the time interval τ that the beam is broken. The car's speed at the photogate location, for a given inclination angle ϕ, is then

$$v_\phi = \frac{w}{\tau_\phi} \tag{8}$$

Having measured the mass of the cart, you can then use equation 5 to calculate its kinetic energy for this inclination angle, so that

$$\Delta K_\phi = \frac{mv_\phi^2}{2} \tag{9}$$

You can measure the inclination angle ϕ with a digital protractor. The gravitational force has the form $F_G = mg$,
so that the change in potential energy corresponding to the inclination angle ϕ is equal to

$$\Delta U_\phi = -mgr\cos\theta = -mgr\sin\phi \tag{10}$$

Since conservation of energy states that $\Delta K = -\Delta U$, we expect that

$$\begin{aligned} -\Delta U &= \Delta K \\ &\Downarrow \\ mgr\sin\phi &= \frac{mv_\phi^2}{2} \end{aligned}$$

Prior to conducting the experiment, you should verify that the function mode is set to *s1*. The function mode can be altered by repeatedly pushing the function button. The device cycles through each of the functions; an LED indicates which function is activated. There is also an LED indicator that identifies the units in which the result will be output on the display panel. You should press the *clear* button, to clear out any previous results.

1. Measure the mass m of the cart.
2. Measure the displacement length r.
3. For inclination angles of $\phi = 10°$, $15°$, and $20°$:
   (a) Measure the final speed v_ϕ and calculate the final kinetic energy $K_\phi = \frac{mv_\phi^2}{2}$.
   (b) Calculate the change in potential energy $\Delta U_\phi = -mgr\sin\phi$.
4. Plot K_ϕ vs. $-\Delta U_\phi$. Does the result seem to verify your expectations?