Physics 201 Lab 8: Finding the Center of Mass

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Theoretical Discussion

The center of mass coordinate of a system or an extended object is defined so that Newton's law of motion, in the form

$$\frac{d\vec{p}}{dt} = \vec{F}_{ext} \tag{1}$$

applies to the sytem or object as if it were a point particle located at the center of mass coordinate and the external force were applied at that point. If we define the center of mass coordinate as

$$\vec{r}_{c.m.} = \frac{\sum_j m_j \vec{r}_j}{\sum_j m_j} = \frac{\sum_j m_j \vec{r}_j}{M_{tot}} \tag{2}$$

where m_j is the mass of the j^{th} particle in the system or object, $\vec{r}_j$ is the position vector of the j^{th} particle, and M_{tot} is the system or object's total mass, we see that the center of mass so defined does indeed satisfy equation 1, since

$$\begin{aligned}
\frac{d\vec{p}}{dt} &= \frac{d}{dt}\left(M_{tot}\vec{v}_{c.m.}\right) \\
&= \frac{d}{dt}\left(M_{tot}\frac{d\vec{r}_{c.m.}}{dt}\right) \\
&= \frac{d}{dt}\left(\sum_j m_j \frac{d\vec{r}_j}{dt}\right) \\
&= \frac{d}{dt}\left(\sum_j \vec{p}_j\right) \\
&= \sum_j \vec{F}_j \\
&= \vec{F}_{ext}
\end{aligned}$$

Since the center of mass coordinate is defined so that the external force can be taken to act at that point, it is constant in time, and the geometric interpretation of Newton's Law for rotational motion (equation 3);

$$\vec{\tau}_{ext} = \frac{d\vec{L}_{c.m.}}{dt} = \frac{d}{dt}\left(\vec{r}_{c.m.} \times \vec{p}_{c.m.}\right) = \vec{r}_{c.m.} \times \frac{d\vec{p}_{c.m.}}{dt} = \vec{r}_{c.m.} \times \vec{F}_{ext} \tag{3}$$

is that the displacement vector $\vec{r}_{c.m.}$ points from the pivot point to the center of mass coordinate. Figure 1 depicts this geometric interpretation. Using the right-hand rule, we see that the torque in this case is negative (into the page), so that the celebrated swingee will rotate clockwise under the influence of gravity.
Since the magnitude of the vector cross product is given by

$$\left|\vec{A} \times \vec{B}\right| = |A||B|\sin\theta$$

where θ is the smallest angle between $\vec{A}$ and $\vec{B}$, then the magnitude of the torque is given by equation 4 below.

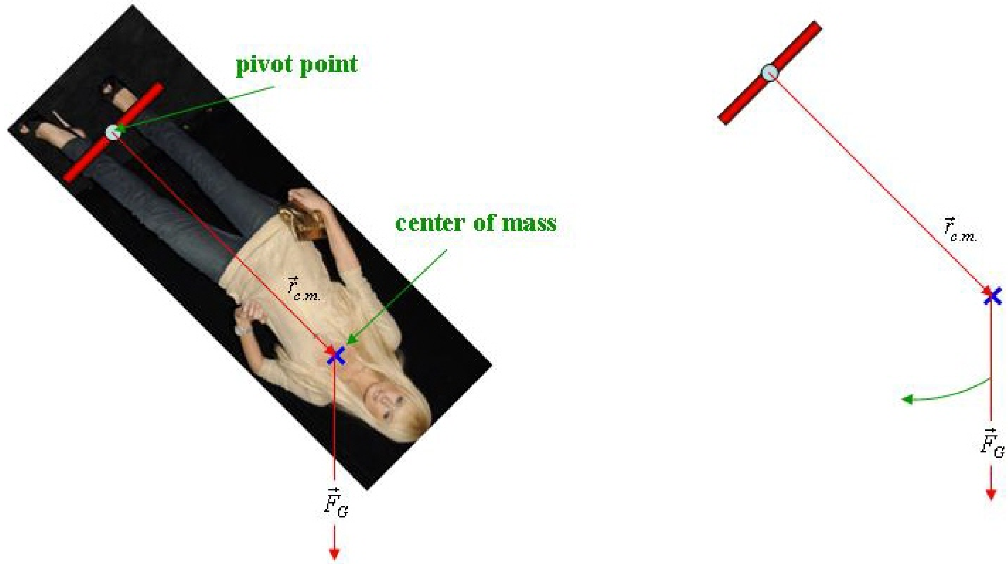

FIG. 1: Relative orientation of the vectors $\vec{r}_{c.m.}$ and $\vec{F}_{ext}$ for a celebrity suspended from a pivot.

$$|\vec{\tau}_{ext}| = |\vec{r}_{c.m.}||\vec{F}_{ext}|\sin\theta \tag{4}$$

Equation 4 makes clear that the torque vanishes whenever the vector $\vec{r}_{c.m.}$ from the pivot point to the center of mass is colinear with the external force $\vec{F}_{ext}$. For an object suspended at rest under its own weight, this implies the following results, which we will make use of in this experiment:

- An object suspended at rest under its own weight is in static equilibrium. Therefore, both the net force and net torque on it are zero.
- Since the torque will only vanish in the gravitational field if $\vec{r}_{c.m.}$ and $\vec{F}_G$ are parallel (or anti-parallel), the object will align itself so that the vector $\vec{r}_{c.m.}$ from the pivot point to the center of mass is *vertical*.

It follows that in suspending a massive object from a variety of different pivot points, all vertical lines originating from the respective pivot points will intersect at the center of mass.

Experimental Procedure

Figure 2 depicts the experimental set-up for this experiment. In brief, you will sequentially suspend the L-bracket from each of three pivot points. In each case, you will trace a vertical line along a plumb bob hanging from the pivot point. The point where all three lines intersect is the center of mass of the L-bracket. Note that the center of mass coordinate of a body need not lie within the body itself.

Summary Procedure

1. Tape a sheet of paper into the inside corner of the L-bracket, as shown in figure 2.
2. For each of the three possible pivot points:
   (a) Hang a plumb bob from the pivot point.
   (b) Mark two points along the plumb line, widely separated enough so that you can later draw an accurate line.

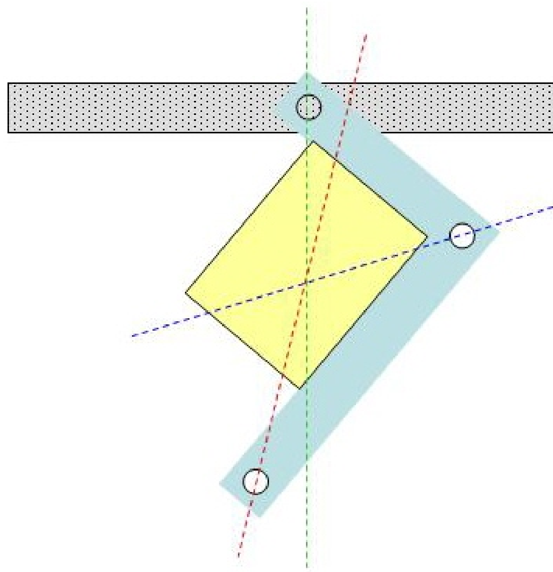

FIG. 2: Experimental scheme for measuring the center of mass.

3. Remove the L-bracket from the hanger and carefully draw in the three lines indicated by your points.
4. The lines should intersect at a single point, or at worst, make a small triangle. In the former case, you have located the center of mass coordinate. In the later case, if all sides of the triangle are smaller than 1 cm, take the center of the triangle as the center of mass coordinate. Otherwise, repeat the measurements.
5. Report your measured center-of-mass for the object.