Physics 101 Lab 8: Torque

Dr. Timothy C. Black

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Theoretical Discussion

For each of the linear kinematic variables; displacement $\vec{r}$, velocity $\vec{v}$ and acceleration $\vec{a}$; there is a corresponding angular kinematic variable; angular displacement $\vec{\theta}$, angular velocity $\vec{\omega}$, and angular acceleration $\vec{\alpha}$, respectively. Associated with these kinematic variables are dynamical variables—momentum and force for linear variables—angular momentum and torque for angular variables. The rotational analogue of the inertial mass m is the moment of inertia I[1].

The angular analogue of force is the torque: Just as force acts to change the magnitude and/or direction of an object's linear velocity, torque acts to change the magnitude and/or direction of an object's angular velocity. The equation

$$\vec{\tau} = I\vec{\alpha} \tag{1}$$

describes the *effect* of a torque on an object's angular kinematic variables. It tells you what a torque does, but not where it comes from. A torque arises whenever a force acts upon a rigid body that is free to rotate about some axis. If the applied force is $\vec{F}$ and the displacement vector from the axis of rotation to the point where the force is applied is $\vec{r}$, then the magnitude of the torque is equal to

$$\tau = rF\sin\theta \tag{2}$$

where θ is the smallest angle between the vectors $\vec{F}$ and $\vec{r}$. The direction of the torque vector is given by the right-hand rule—place the fingers of your right hand along $\vec{r}$ and curl them into $\vec{F}$: your thumb will point in the direction of $\vec{\tau}$. The directional relationships between $\vec{F}$, $\vec{r}$, and $\vec{\tau}$ are shown in figure 1.

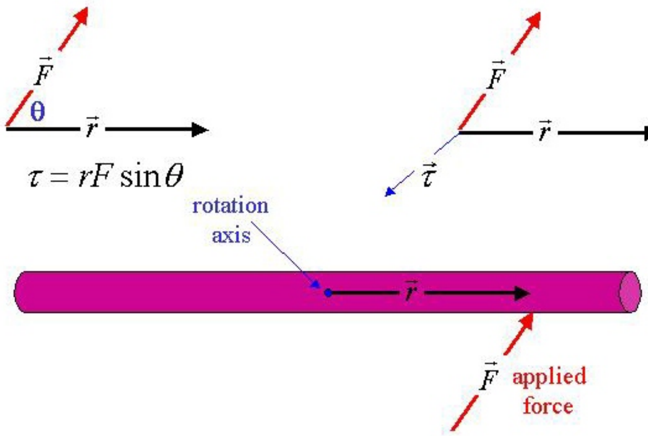

FIG. 1: Directional relationships between $\vec{F}$, $\vec{r}$, and $\vec{\tau}$

So long as the total net force on an object is zero, the velocity of its center of mass will not change. However, it is possible for an object to have zero net force acting on it, but to nevertheless have a non-zero torque acting on it. Figure 2 shows one such possible scenario. The velocity of the center of mass of the object in figure 2, acted upon by two equal and opposite forces, will remain constant, but since the torque is non-zero, it will spin about its axis at an ever-increasing rate of rotation.

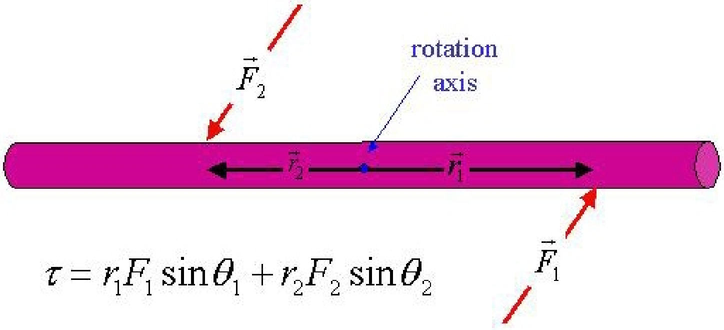

FIG. 2: Equal forces applied to opposite sides of a rotation axis

For an object to remain in *static equilibrium*, so that both the velocity of its center of mass and its angular velocity about any axis are constant, both the net force and the net torque on it must equal zero. The conditions for static equilibrium are:

$$\vec{F}_{\text{net}} = 0 \quad (3)$$
$$\vec{\tau}_{\text{net}} = 0 \quad (4)$$

In today's lab you will use the conditions for static equilibrium to measure the mass of a meter stick that is balanced on a knife-edge fulcrum.

PROCEDURE

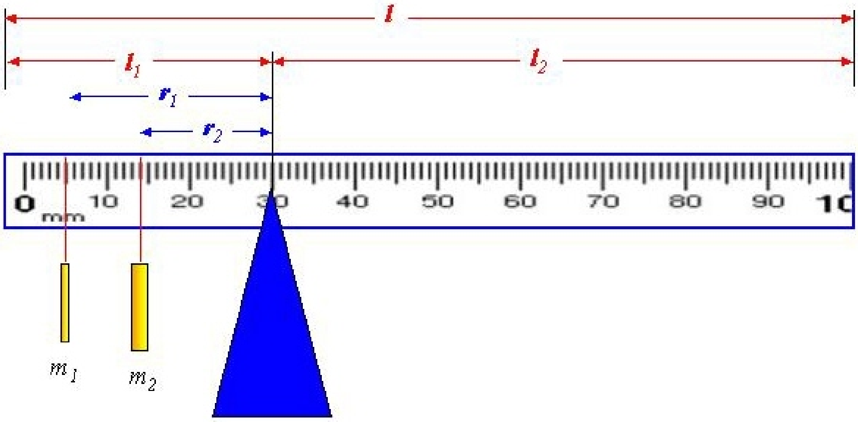

FIG. 3: Experimental setup for using torque to weigh a meter stick

The experimental setup is shown in figure 3. Suppose you can get your meter stick in balance as shown in the figure. There are three forces acting on the left-hand side of the stick and one force acting on the right-hand side. The mass of the section of ruler on the right hand side is $m_{rhs} = m\frac{l_2}{l}$, where $l = 100$ cm is the length of the ruler. The center of mass of the ruler material on the right-hand side is located a distance $\frac{l_2}{2}$ from the fulcrum, as illustrated in figure 4. Since the force acts at right angles to the displacement, the magnitude of the total torque acting on the right-hand side is

$$\tau_{\text{right}} \;=\; m_{rhs} g \frac{l_2}{2} = mg\frac{l_2^2}{2l} \tag{5}$$

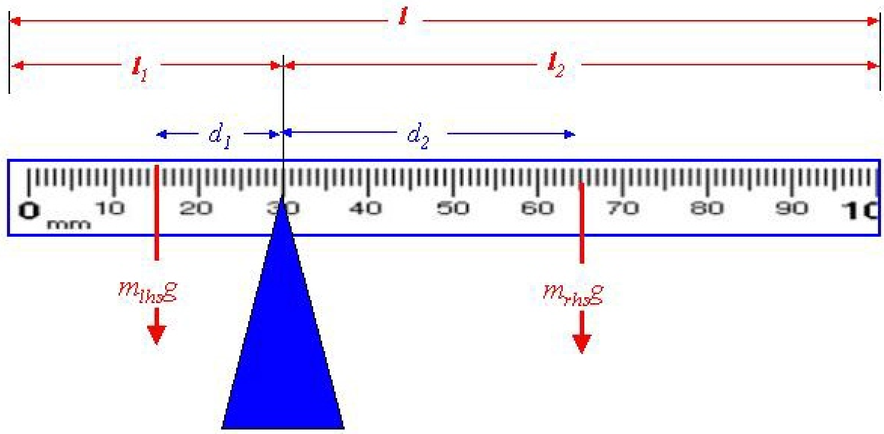

FIG. 4: Illustration of the balance of torques

On the left hand side, there are the gravitational forces due to m_1, m_2, and the mass of the ruler material m_{lhs} on the left-hand side of the stick. The magnitude of the forces arising from these three sources are, respectively,

$$\begin{aligned} F_1 &= m_1 g \\ F_2 &= m_2 g \\ F_3 &= m_{lhs} g = m\frac{l_1}{l} g \end{aligned}$$

The center-of mass of the ruler material on the left-hand side is located a distance $\frac{l_1}{2}$ from the fulcrum. Since all three forces act at right angles to the displacement, the total torque on the left-hand side is

$$\tau_{\text{left}} \;=\; m_1 g r_1 + m_2 g r_2 + mg\frac{l_1^2}{2l} \tag{6}$$

All of the downward forces acting on the ruler are countered by an equal and opposite upward reaction force (normal force) that acts at the point of the fulcrum. Since it acts at the fulcrum point, it exerts no torque, so that the equation for static equilibrium is

$$\tau_{\text{left}} \;=\; \tau_{\text{right}} \longrightarrow m_1 g r_1 + m_2 g r_2 + mg\frac{l_1^2}{2l} = mg\frac{l_2^2}{2l} \tag{7}$$

Cancelling the common factor of g, and re-arranging the equation, you get an equation for the mass of the ruler:

$$m = \frac{2l(m_1 r_1 + m_2 r_2)}{(l_2^2 - l_1^2)} \tag{8}$$

By varying the fulcrum point, and hence the values of l_1 and l_2, and adjusting the locations of m_1 and m_2 to achieve static equilibrium, you can obtain independent measurements of the mass of the ruler.

Summary

1. Set the fulcrum location at approximately 30 cm.
2. Bring the meter stick into balance (static equilibrium) by varying the positions of the two masses, m_1 and m_2. The inner mass m_2 (mass closest to the fulcrum) should be about 150 g. The outer mass m_1 should be about 10–20 g. You can use the position of m_1 to "fine-tune" the balance.
3. Record the distances r_1, r_2, l_1, l_2, m_1 and m_2.
4. Calculate the ruler's mass using the method of static equilibrium (equation 8) and label it m_{exp}.
5. Weigh the ruler using the mass balance. Record this value as m_b.
6. Calculate and report the fractional discrepancy δ between m_{exp} and m_b.

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[1] The moment of inertia I for a collection of masses m_j, is $I = \sum_j r_j^2 m_j$, where the displacement r_j is from the axis of rotation to the location of the mass m_j. For the case of continuous bodies, the expression is more complicated and requires integral calculus to evaluate.