Calculus-based Physics 201

Test #2:

Force, Newton's Laws, Work, Kinetic & Potential Energy, Conservation of Energy

Fall 2024

Numerical Answers: Use the MKS Metric System unless explicitly told to do otherwise.
Give Numerical Answers to at least Three Significant Figures.

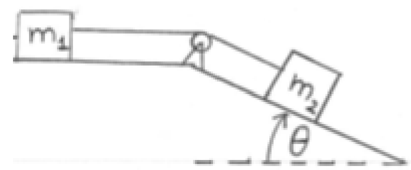
NO: Notes, Formula Cards, Formulas or Constants in your Calculators, or any other aids.
Vectors are indicated in Bold Print (**v**).

Unit Vectors in the x, y, & z directions are symbolized by **i**, **j**, & **k** respectively.
Multiple Choice are 5 Points each. Problem total to 40 Points

Chapt 5/6: F = ma incline plane with kinetic friction:

| 1> A block of mass $m_1 = M$ lies on a surface and is connected with a light cord which runs over a massless and frictionless pulley to a block of mass $m_2 = 2M$ which lies on an inclined plane with inclination angle $\theta = 30°$. If the coefficient of Kinetic Friction between the blocks and the surfaces on which they slide is 0.20, then what is the acceleration of the blocks in m/s^2?

->a) 1.5 b) 1.7 c) 1.9 d) 2.2 e) 5.1



Component of the Equations of Motion Perpendicular to the Incline or Surface:

$F(\text{net on } m_1) = m_1 0 \Rightarrow N_1 - m_1 g = 0 \Rightarrow N_1 = m_1 g$

$F(\text{net on } m_2) = m_2 0 \Rightarrow N_2 - 2Mg\cos\theta = 0 \Rightarrow N_2 = 2Mg\cos\theta$

Component of the Equations of Motion along the Incline or Horizontal Surface:

$F(\text{net on } m_1) = m_1 a \Rightarrow T - (f_k = \mu_k N_1 = \mu_k Mg) = Ma$ &

$F(\text{net on } m_2) = m_2 a \Rightarrow 2Mg\sin\theta - T - (f_k = \mu_k N_2 = \mu_k 2Mg\cos\theta) = 2Ma$; add these Eq^ns

$T - \mu_k Mg + 2Mg\sin\theta - T - \mu_k(2Mg\cos\theta) = Ma + 2Ma$

$2Mg\sin\theta - \mu_k(2Mg\cos\theta) - \mu_k Mg = 3Ma \Rightarrow a = g\{2\sin\theta - 2\mu_k\cos\theta - \mu_k\}/3$

$a = g\{2\sin\theta - 2\mu_k\cos\theta - \mu_k\}/3 = g(1 - 0.2\sqrt{3} - 0.2)/3 = 0.4536g/3 = 1.481726806$

Chapt 5: Newton's 3rd Law:

| 2> A Physics 201 Student rides an elevator upward at a constant speed. Consider the following four forces that arise in this situation:

$\mathbf{F}_1$ = the weight of the elevator
$\mathbf{F}_2$ = the weight of the Student
$\mathbf{F}_3$ = the force of the Student pulling on the Earth
$\mathbf{F}_4$ = the force of the elevator floor pushing on the Student

Which two forces form an "Action- Reaction" pair that obeys Newton's 3rd Law

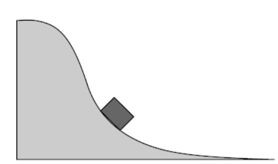
a) 1 & 2 ->b) 2 & 3 c) 1 & 3 d) 2 & 4 e) 3 & 4

Earth exerts a Force on the Student and the Student exerts an equal and oppositely directed Force on the Earth.

Chapt. 5: Force along an Incline affects acceleration along the incline:

| 3> A block on a curved surface slides without friction down the track shown in the Figure. As the block slides beyond the point shown, what happens to its speed and acceleration in the direction of motion (ie the tangential direction)?

a) Need to know the Radius of Curvature of the track at the Block's location.
b) Both decrease. c) Both increase.
d) The speed decreases, but the tangential acceleration increases.
->e) The speed increases, but the tangential acceleration decreases.

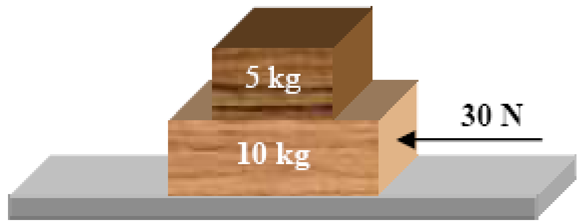


The Block is at a Point of Inflection where the slope begins to decrease where before it was increasing: the component of the force of gravity along the incline is mgsinθ where θ is the slope of the incline at a particular point (slope of the tangent line). The tangential acceleration is $a_t = dv/dt = g\sin\theta$ (the centripetal acceleration v^2/ρ_c is not relevant to a change in speed, it is only relevant to a change in direction of motion). This tangential acceleration in the direction of motion decreases. The speed continually increases at the rate of gsinθ.

Chapt 6: F = ma used twice w/Static friction between two blocks:

| 4> Two blocks slide across a horizontal tabletop as shown. The surface between the top and bottom blocks is roughened so that there is no slipping between them. A 30.0 N force applied to the bottom block as shown causes the blocks to accelerate to the left with an acceleration of 1.00 m/s^2. What is the coefficient of friction between the bottom block and the tabletop.

->a) 0.102 b) 0.136 c) 0.153 d) 0.204 e) 0.306



For the two block system in the Vertical Direction: $F_{net\_vertical} = N - (m + M)g = 0$
=> $N = (m + M)g$

For the two block system in the Horizontal Direction: $F_{net\_horizontal} = M_{total}a$ =>
$F_{applied} - [f_k = \mu_k N = \mu_k(m + M)g] = (m + M)a$ => $F_{applied} - \mu_k(m + M)g] = (m + M)a$
=> $\mu_k = [F_{applied} - (m + M)a]/(m + M)g] = [30 - 15(1)/(15g) = 1/g = 0.1020408163$

Chapt 6: F = ma Uniform Circular Motion:

| 5> A m = 3.00 kg mass attached to a light string revolves in circular motion on a horizontal, frictionless table. The radius of the circle is 0.800 m. The string can support a mass of M = 25.0 kg (hanging vertically) before breaking. What maximum speed in m/s can the mass have before the string breaks?

a) 1.08 b) 7.23 ->c) 8.08 d) 14.0 e) 65.3

Maximum T via: T = Mg; This Tension is the Centripetal Force in the UCM part.

$\mathbf{F}_c = m\mathbf{a}_c$ where $a = v^2/r$ => $(T\mathbf{=}Mg) = mv^2/r$ => $v = \sqrt{[(M/m)rg]} = \sqrt{[(25/3)0.8*9.8]}$ =
v = 8.082903769

Chapt. 7: Work done by a constant Force (simple):

| 6> A 5.0 kg object is pulled along a horizontal surface at a constant speed by a 15 N force acting 20° above the horizontal. How much work in joules is done by this constant force as the object moves 6.0 m?

a) 0 b) 31 c) 74 ->d) 85 e) 90

Since **F** is Constant: $W = \mathbf{F.d} = 15(6)\cos 20 = 84.57233587$

Chapt. 7: Power:

| 7> If a car generates 15 horsepower (1 horsepower ≡ 746 Watts) when travelling in a straight line at a steady speed of 90 km/hr, what must be the net force in Newtons exerted on the car due to the combined effects of resistive forces including air drag?

a) 0.60 N b) 12.4 N c) 330 N d) 375 N ->e) 448 N

NOTE: $\mathbf{F}_{net\_on\_car}\cdot\mathbf{v} = \Delta K = 0$

$\mathbf{F}_{net} = \mathbf{F}_{static\_pushing\_car} + \mathbf{F}_{Resistive} = m\mathbf{a} = 0$ => $\mathbf{F}_{static\_pushing\_car} = -\mathbf{F}_{Resistive}$ => $F_{pushing} = F_{Resistive}$

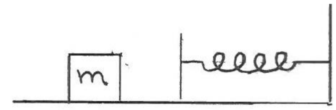
90 km/hr(hr/3600s)(1,000m/km) = 25.00000000 m/s

$\mathbf{F}_{static\_pushing\_car}\cdot\mathbf{v} = Fv\cos 0$ = Power = 15(746) Watts => F = Power/v = 447.6000000 N

Chapt. 7 or 8: Work-Energy or Conservation of Mechanical Energy with a Spring:

| 8> A 2.0 kg block slides on a horizontal frictionless surface. It is heading straight for the free end of a spring with spring constant k = 2,000 N/m; the other end is attached to the wall. The speed of the block before it touches the spring is 6.0 m/s. How fast is the block moving in m/s at the instant the spring has been

compressed by 15 cms?
->a) 3.7 b) 4.4 c) 4.9 d) 5.4 e) 7.6



$W_{net} = (W_s = ½kx_i^2 - ½kx_f^2) + (W_g = m\mathbf{g}\cdot\mathbf{d} = 0) + (W_N = \mathbf{N}\cdot\mathbf{d} = 0)$
$W_{net} \equiv \Delta K \Rightarrow K_f - K_i = ½kx_i^2 - ½kx_f^2 \Rightarrow K_f = K_i + ½k(x_i = 0)^2 - ½k(x_f = -0.15)^2$
$\Rightarrow ½(m = 2)v_f^2 = ½(m = 2)(v_i = 6)^2 - ½(k=2,000)(x_f = -0.15)^2$
$\Rightarrow v_f^2 = 36 - 1,000(0.02250000) = 36 - 22.5 = 13.5 \Rightarrow v_f = 3.674234614$ m/s
Chapt. 7 above & Chapt. 8 below
$W_{ext} = \Delta E_{sys} = \Delta E_{mech} + (\Delta E_{thermal} = 0) + (\Delta E_{internal} = 0)$
$\Rightarrow 0 = \Delta K + \Delta U_g + \Delta U_s = ½mv_f^2 - ½mv_i^2 + ½kx_f^2 - ½kx_i^2 \Rightarrow ½mv_f^2 = ½mv_i^2 - ½kx_f^2 + ½kx_i^2$
$v_f = \sqrt{[2\{½mv_i^2 - ½kx_f^2 + ½kx_i^2\}/m]} = \sqrt{[\{mv_i^2 - kx_f^2 + k0^2\}/m]} = \sqrt{[v_i^2 - kx_f^2/m]}$
$= 3.674234614$

Chapt. 7 or 8: Net-Work Kinetic-Energy Theorem or Work-Energy Theorem:

| 9> A 1.5 kg object moving along the x-axis has a velocity of +4.0 m/s at x=0. If the only force acting on this object is given by $\mathbf{F} = F_x\mathbf{i}$ where,

$$F_x = -4.0x + 8,$$

then what is the kinetic energy (in Joules) of the object at x = +3.0 m?

a) The object could never get to x = +3.0 m because its Kinetic Energy would be *Negative* there!
->b) 18 c) 21 d) 23 e) 26

$W_{net} = W_F \equiv \Delta K \Rightarrow K_f - K_i = 6$ (see below) =>
$\Rightarrow K_f = K_i + W_F = ½(m = 1.5)(v_i = 4)^2 + 6 = [(3/4)16] + 6 = 12 + 6 = 18$
Chapt. 7 above & Chapt. 8 below
$W_{ext} = \Delta E_{sys} = \Delta E_{mech} + \Delta E_{thermal} + \Delta E_{internal} \Rightarrow W_{ext} = \Delta K + \Delta U = ½mv_f^2 - ½mv_i^2$
$W_{ext} = W_F = \int\mathbf{F}.d\mathbf{r} = \int_i^f F_x dx = = \int_i^f(-4x+8)dx = [-4x^2/2 + 8x]_i^f = [-2*3^2+8(3)] - [0]$
$= -18 + 24 = +6 \Rightarrow ½mv_f^2 = W_{ext} + ½mv_i^2 = 6 + 1/2(3/2)16 = 6 + 12 = 18$

Chapt. 7 or 8: Work done by the Drag Force:

|10> A 10 kg object is dropped from rest. After falling a distance of 50 m, it has a speed of 26 m/s. How much work is done by the dissipative air drag force on the object during this descent (in kiloJoules)?

a) -1.3 ->b) -1.5 c) -1.8 d) -2.0 e) -8.3

$W_{net} = W_{Drag} + W_{Grav} \equiv \Delta K \Rightarrow W_{Drag} + m\mathbf{g}\cdot\mathbf{d} = K_f - K_i \Rightarrow W_{Drag} + mgd\cos 0° = K_f - 0$
$\Rightarrow W_{Drag} = -mgd + ½mv_f^2 = -10(9.8)(50) + ½10(26)^2 = -1,520$ J $= 1.52$ kJ
Chapt. 7 above & Chapt. 8 below
$W_{net\_ext} = \Delta E_{sys} = \Delta E_{mech} + \Delta E_{thermal} + \Delta E_{internal} \Rightarrow$
System = Mass + Earth & Air is external: $W_{drag} = \Delta K + \Delta U = ½mv_f^2 - ½mv_i^2 + mg\Delta y$
System = Mass + Earth + Air: $0 = \Delta K + \Delta U + (\Delta E_{thermal} - W_{drag}) + (\Delta E_{internal} = 0)$
$W_{drag} = \Delta K + \Delta U = ½mv_f^2 - ½mv_i^2 + mg\Delta y = ½10(26)^2 - 0 + 10(9.8)(-50) = 3,380 - 4,900 =$
$= -1,520$

Chapt. 8: Internal Energy Changes:

|11> A girl of mass 45.00 kg on ice skates pushes herself away from a wall and as a result glides on the frictionless ice at a constant speed of 2.200 m/s. Therefore, the ...

a) force of the wall on the girl does 108.9 J of work.
b) change in the girl's internal energy is equal to -108.9 J.
->c) change in the girl's internal energy is much less than -108.9 J.
d) change in the girl's internal energy is much greater than -108.9 J.
e) change in the girl's thermal energy is equal to -108.9J.

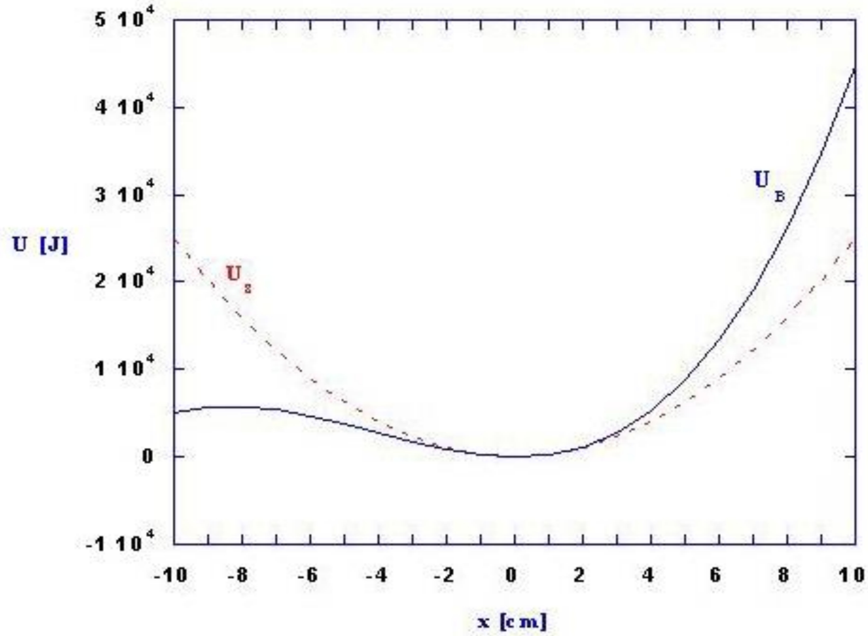
$K_i = 0$ & $K_f = ½mv_f^2 = 108.900$ J $\Rightarrow \Delta K = 108.9$ & System = Girl + Earth + Wall + Ice
$W_{ext} = \Delta E_{sys} = \Delta E_{mech} + \Delta E_{thermal} + \Delta E_{internal} \Rightarrow ...$
$0 = \Delta E_{sys} = (\Delta K = 108.9) + (\Delta U_g = 0) + (\Delta E_{thermal} > 0) + (\Delta E_{internal} < 0)$
$\Delta E_{internal} = -(\Delta E_{thermal} > 0) - \Delta K = -$(LARGE POSITIVE #) $- 108.9 \lll -108.9$

Chapt. 8: Potential Energy Curve:

|12> The figure below shows two potential energy curves; one for an object attached to a spring (U_S, dashed curve), and one for an identical object attached to a bungee cord (U_B, solid curve). Stretching of the spring or the bungee cord is described by x

values that are greater than zero. If the total energy of both systems is 20 kJ, which one stretches the least during the course of the motion?

a) Both the spring and the bungee cord stretch the same amount.
b) This question cannot be answered without more information.
c) The spring.
->d) The bungee cord.



Imagine the horizontal line E(total) = 20 kJ in the above Graph.
The Positive-x Turning Point for the Spring is larger than that for the Bungee Cord.
Therefore the Spring stretches the most and the Bungee Cord the least.

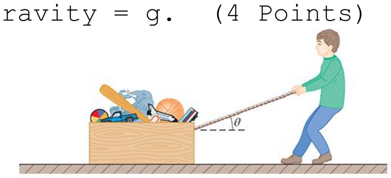
F24> P.6.30: **Optimally pulling a Cart:**

|P1> A toy chest and its contents have a mass m. The coefficient of static friction between toy chest and floor is μ_s. The child attempts to move the chest across the floor by exerting a force, **F**, on the rope that makes an angle θ with respect to the horizontal. The questions below ask you to determine the minimum force required to put the chest on the verge of moving. This minimum force will obviously be a function of θ. You will then find which angle, θ, will yield an Ultimate Minimum Force to put the chest on the verge of moving. To carry out these calculations, use a rectangular xy-coordinate system with a horizontal x-axis increasing to the right, and a vertical y-axis increasing in the upward direction.

a) Write out The Equation of Motion for the chest in the y-direction and solve it for the magnitude of the Normal Force N acting on the cart by the floor. Use symbols: F, θ, m, μ_s, N, and the acceleration due to gravity = g. (4 Points)

$F_{net,y} = ma_y = 0 \Rightarrow N + F\sin\theta - mg = 0 \Rightarrow N = mg - F\sin\theta$

b) Write out The Equation of Motion for the chest in the x-direction and solve it for the magnitude of the force F applied to the rope by the child. Use symbols: F, θ, m, μ_s, N, and the acceleration due to gravity = g. (4 Points)



The chest is on the Verge of Moving => $f_s = f_{s,max} = \mu_s N$

$F_{net,x} = ma_x = \Rightarrow F\cos\theta - [f_s = \mu_s N = \mu_s(mg - F\sin\theta)] = 0$

$F\cos\theta - \mu_s(mg - F\sin\theta) = 0 \Rightarrow F(\cos\theta + \mu_s\sin\theta) = \mu_s mg$

$F = \mu_s mg/(\cos\theta + \mu_s\sin\theta)$

c) Find the angle θ_o that yields the Ultimate Minimum Force, $\mathbf{F}_{min}$, that will put the chest on the verge of moving. Use symbols as usual. (4 Points)

$dF/d\theta = [(\cos\theta + \mu_s\sin\theta)0 - \mu_s mg(-\sin\theta + \mu_s\cos\theta)]/(\cos\theta + \mu_s\sin\theta)^2 = 0$

$\Rightarrow \mu_s mg(-\sin\theta + \mu_s\cos\theta) = 0 \Rightarrow \sin\theta = \mu_s\cos\theta \Rightarrow \tan\theta_o = \mu_s \Rightarrow \theta_o = \tan^{-1}(\mu_s)$

d) What is this F_{min} (in your most simplified form)? Use symbols as usual. (5 Points)

$F_{min} = \mu_s mg/(\cos\theta_o + \mu_s\sin\theta_o) = \tan\theta_o mg/(\cos\theta_o + \tan\theta_o\sin\theta_o)$

$= mg\sin\theta_o/[(\cos\theta_o + \sin\theta_o^2/\cos\theta_o)\cos\theta_o] = mg\sin\theta_o$ ... WOW! or do it this way ...

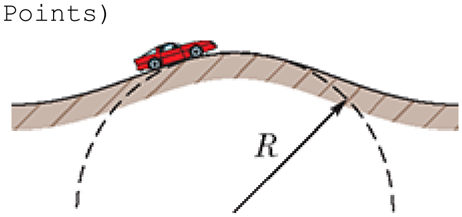
If $\tan\theta_o = \mu_s$ in a Right Triangle, then the opposite side to θ_o is μ_s and the adjacent side is 1 => hypotenuse = $\sqrt{[1 + \mu_s^2]}$ => $\sin\theta_o = \mu_s/\sqrt{[1 + \mu_s^2]}$ and $\cos\theta_o = 1/\sqrt{[1 + \mu_s^2]}$

$\Rightarrow F_{min} = \mu_s mg/(\cos\theta_o + \mu_s\sin\theta_o) = \mu_s mg/(1/\sqrt{[1 + \mu_s^2]} + \mu_s^2/\sqrt{[1 + \mu_s^2]}) =$

$= \mu_s mg\sqrt{[1 + \mu_s^2]}/(1 + \mu_s^2) = \mu_s mg/\sqrt{[1 + \mu_s^2]} = mg\sin\theta_o$ ... WOW!

F24> P.6.82

|P2> A stuntman drives a car (without negative lift) over the top of a hill, the cross section of which can be approximated by a circle of radius R. What is the Greatest Speed, v, at which he can drive without the car leaving the road at the top of the hill? Use defined Symbols. (8 Points)



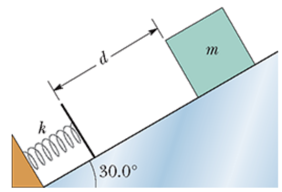
Let vectors pointing to the Center of Curvature of the road be positive (like **a**).

$F_{rad} = ma_{rad} \Rightarrow -N + mg = mv^2/R$ When N = NormalForce = 0 he is on the verge of leaving the road => $mg = mv^2/R \Rightarrow v^2 = gR \Rightarrow v = \sqrt{[gR]}$

F24> P.8.129 **Block sliding along an inclined plane onto a Spring:**

|P3> A block with mass m = 3.20 kg starts from rest and slides distance d down a frictionless incline, where it runs into a spring with spring constant k = 431 N/m. The incline makes angle θ = 30° with respect to the horizontal. The block slides an additional distance x_{max} = 21.0 cm before it is brought to rest momentarily by compressing the spring. The distance the spring is compressed from equilibrium is the variable x (x is NOT the total distance that the block moves after its release).

a) Use Energy Methods to analyze the Motion (from the moment the block is released until the moment when the block is brought to rest momentarily by compressing the spring), and use the result of this analysis to obtain an Equation for d. Do this using ONLY the SYMBOLS defined above: m, d, k, θ, x, x_{max}, and the acceleration due to gravity, g. DO NOT insert values for these Symbols (such as θ = 30° as shown in the Diagram)! (5 Points)



System = Block, Incline, Spring, and Earth. i = initial = moment of release, and f = final = moment of maximum compression of spring.

$W_{ext} = \Delta E_{sys} = \Delta E_{mech} + (\Delta E_{thermal} = 0) + (\Delta E_{internal} = 0) = \Delta K + \Delta U_g + \Delta U_s$

$0 = \Delta E_{mech} = ½m(v_f = 0)^2 - ½m(v_i = 0)^2 + \Delta(mgy) + ½k(x_f = x_{max})^2 - ½k(x_i = 0)^2$

$0 = mg[\Delta y = -(d + x_{max})\sin\theta] + ½kx_{max}^2 \Rightarrow mg(d + x_{max})\sin\theta] = ½kx_{max}^2$

$d = (½kx_{max}^2 - mgx_{max}\sin\theta)/(mg\sin\theta) = (kx_{max}^2/(2mg\sin\theta) - x_{max}$

b) What is the Kinetic Energy, K, of the block at x values between x = 0 & x = x_{max}? Do all work in SYMBOLS as explained in Part (a)! (5 Points)

System = Block, Incline, Spring, and Earth. i = initial = moment of release, and f = final = arbitrary moment after first contact with spring & before maximum compression.

$W_{ext} = \Delta E_{sys} = \Delta E_{mech} + (\Delta E_{thermal} = 0) + (\Delta E_{internal} = 0) = \Delta K + \Delta U_g + \Delta U_s$

$0 = \Delta E_{mech} = ½m(v_f = v)^2 - ½m(v_i = 0)^2 + \Delta(mgy) + ½k(x_f = x)^2 - ½k(x_i = 0)^2$

$0 = ½mv^2 + mg[\Delta y = -(d + x)\sin\theta] + ½kx^2 \Rightarrow K = ½mv^2 = mg(d + x)\sin\theta - ½kx^2$

$K = ½mv^2 = mg(d + x)\sin\theta - ½kx^2$

c) What is the distance x = x_o, between the point of first contact with the spring and the point where the block's speed (or Kinetic Energy!) is greatest? Do all work in SYMBOLS as explained in Part (a)! (5 Points)

We have the Kinetic Energy as a function of x in Part (b):

$K(x) = mg(d + x)\sin\theta - ½kx^2$

K is a maximum or a minimum when dK/dx = 0.

$dK/dx = 0 + mg\sin\theta - ½k2x = 0 \Rightarrow x = x_o = mg\sin\theta/k$

BONUS> What is the Physical Reason for why the Kinetic Energy is a maximum at x = x_o? What are the numerical values for d and x_o? (5 Points)

x_o is the point of Equilibrium for m on the incline since mgsinθ = kx (where the spring force is balanced by the component of the gravitational force down the incline) as we discussed in class for a similar problem(s).

$\mathbf{F}_{net} = m\mathbf{a}$ => the acceleration down the incline = a = (mgsinθ - kx)/m and a > 0 for x < x_o. This is why K is a maximum, because K increases for 0 < x < x_o.

$d = (kx_{max}^2/(2mg\sin\theta) - x_{max} = 0.6060937500 - 0.21 = 0.3960937500$ m

$x = x_o = mg\sin\theta/k = 0.03638051044$ m = 3.64 cm