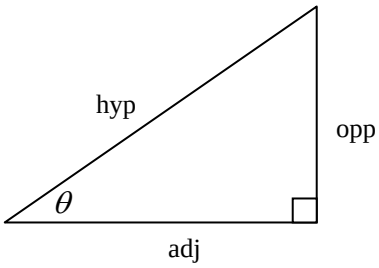
Trigonometric Relationships and Values

$$\sin(\theta)=\frac{opp}{hyp} \qquad \csc(\theta)=\frac{hyp}{opp}$$

$$\cos(\theta)=\frac{adj}{hyp} \qquad \sec(\theta)=\frac{hyp}{adj}$$

$$\tan(\theta)=\frac{opp}{adj} \qquad \cot(\theta)=\frac{adj}{opp}$$



$$\csc(\theta)=\frac{1}{\sin(\theta)} \qquad \sec(\theta)=\frac{1}{\cos(\theta)} \qquad \tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta)=\frac{1}{\tan(\theta)}=\frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities

$$\sin^2(\theta)+\cos^2(\theta)=1$$
$$\tan^2(\theta)+1=\sec^2(\theta)$$
$$1+\cot^2(\theta)=\csc^2(\theta)$$

Sum and Difference Identities

$$\sin(\theta+\phi)=\sin(\theta)\cos(\phi)+\cos(\theta)\sin(\phi)$$
$$\sin(\theta-\phi)=\sin(\theta)\cos(\phi)-\cos(\theta)\sin(\phi)$$
$$\cos(\theta+\phi)=\cos(\theta)\cos(\phi)-\sin(\theta)\sin(\phi)$$
$$\cos(\theta-\phi)=\cos(\theta)\cos(\phi)+\sin(\theta)\sin(\phi)$$
$$\tan(\theta+\phi)=(\tan(\theta)+\tan(\phi))/(1-\tan(\theta)\tan(\phi))$$
$$\tan(\theta-\phi)=(\tan(\theta)-\tan(\phi))/(1+\tan(\theta)\tan(\phi))$$

Double Angle Identities

$$\sin(2\theta)=2\sin(\theta)\cos(\theta)$$
$$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)=2\cos^2(\theta)-1=1-2\sin^2(\theta)$$
$$\tan(2\theta)=2\tan(\theta)/(1-\tan^2(\theta))$$

Half Angle Identities

$$\sin^2(\theta)=\tfrac{1}{2}(1-\cos(2\theta))$$
$$\cos^2(\theta)=\tfrac{1}{2}(1+\cos(2\theta))$$

Product to Sum Identities

$$\sin(A)\cos(B)=\tfrac{1}{2}[\sin(A-B)+\sin(A+B)]$$
$$\sin(A)\sin(B)=\tfrac{1}{2}[\cos(A-B)-\cos(A+B)]$$
$$\cos(A)\cos(B)=\tfrac{1}{2}[\cos(A-B)+\cos(A+B)]$$

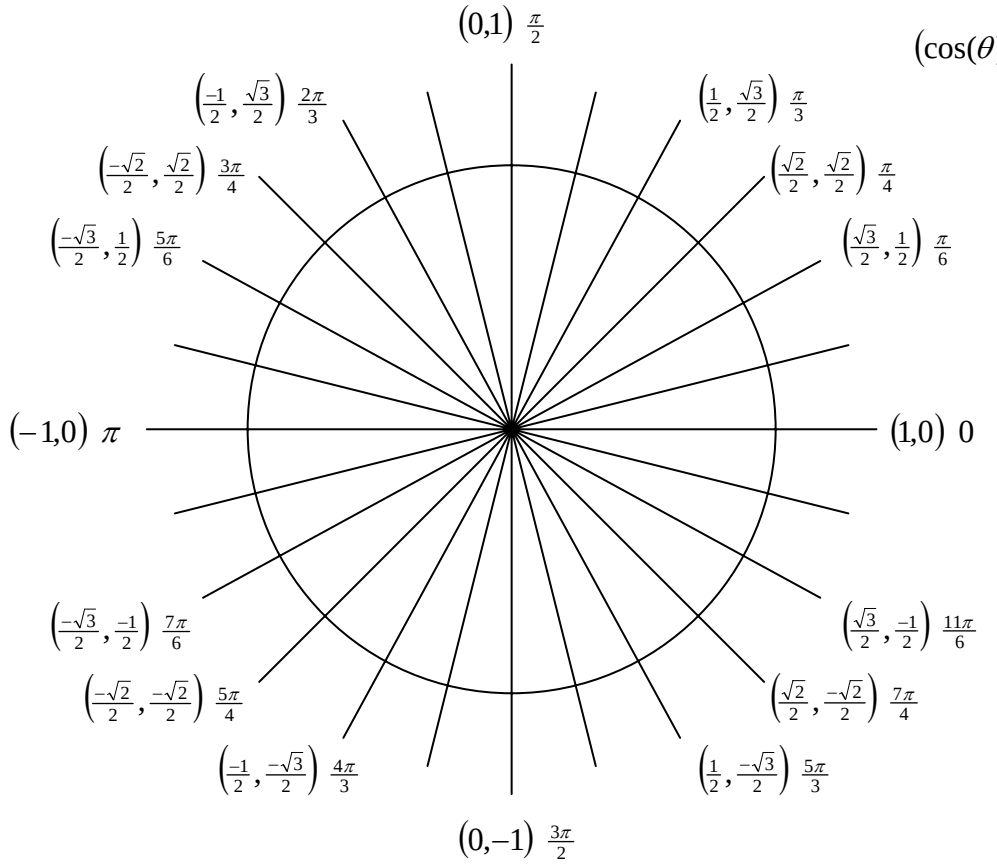
Negative Argument Identities

$$\sin(-\theta)=-\sin(\theta)$$
$$\cos(-\theta)=\cos(\theta)$$
$$\tan(-\theta)=-\tan(\theta)$$

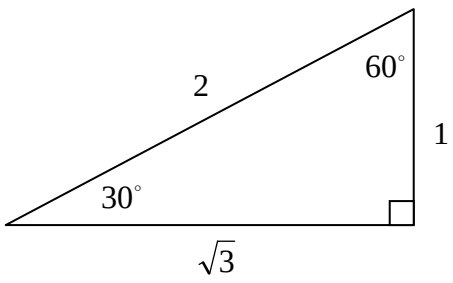
$\frac{\pi}{2}$ Shifts

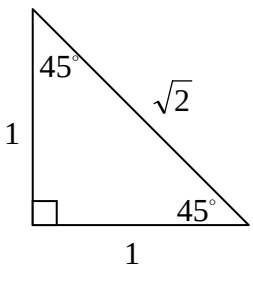
$$\sin(\theta-\tfrac{\pi}{2})=-\cos(\theta) \qquad \cos(\theta-\tfrac{\pi}{2})=\sin(\theta) \qquad \tan(\theta-\tfrac{\pi}{2})=\tan(\theta)$$

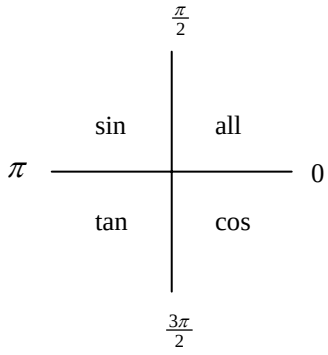
$$\sin(\tfrac{\pi}{2}-\theta)=\cos(\theta) \qquad \cos(\tfrac{\pi}{2}-\theta)=\sin(\theta) \qquad \tan(\tfrac{\pi}{2}-\theta)=\cot(\theta)$$



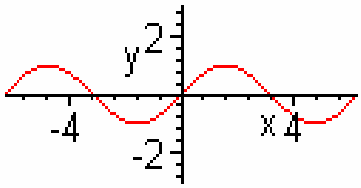
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	U	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	U	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$





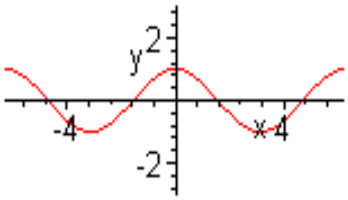


$f(x) = \sin(x)$



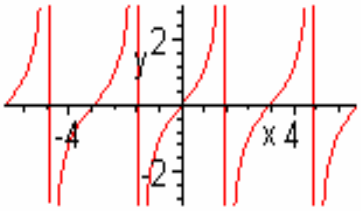
domain: $-\infty \le x \le \infty$
range: $-1 \le y \le 1$

$f(x) = \cos(x)$



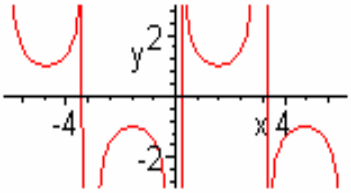
domain: $-\infty \le x \le \infty$
range: $-1 \le y \le 1$

$f(x) = \tan(x)$



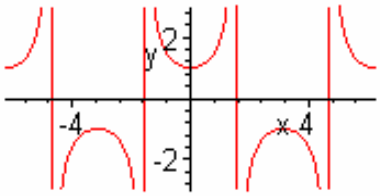
domain: all real #s except odd integer multiples of $\frac{\pi}{2}$
range: $-\infty \le y \le \infty$

$f(x) = \csc(x)$



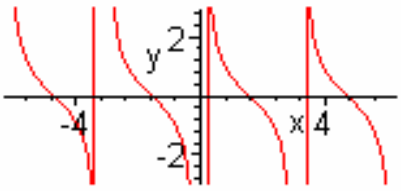
domain: all real #s except even integer multiples of $\frac{\pi}{2}$
range:
$-\infty < x \le -1, 1 \le x < \infty$

$f(x) = \sec(x)$



domain: all real #s except even integer multiples of $\frac{\pi}{2}$
range:
$-\infty < x \le -1, 1 \le x < \infty$

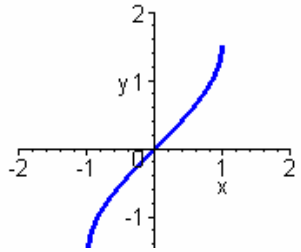
$f(x) = \cot(x)$



domain: all real #s except odd integer multiples of $\frac{\pi}{2}$
range: $-\infty \le y \le \infty$

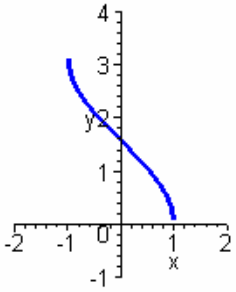
Inverse Trigonometric Functions

$f(x) = \arcsin(x)$



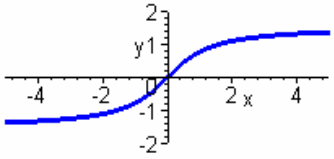
domain: $-1 \le x \le 1$
range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$f(x) = \arccos(x)$



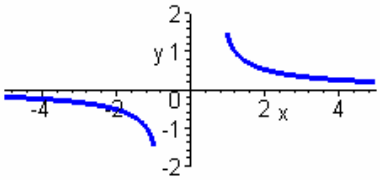
domain: $-1 \le x \le 1$
range: $0 \le y \le \pi$

$f(x) = \arctan(x)$



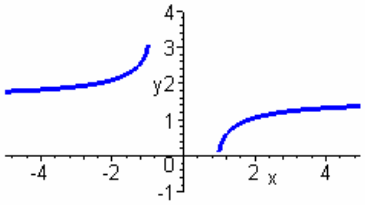
domain: $-\infty < x < \infty$
range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$f(x) = arc\csc(x)$



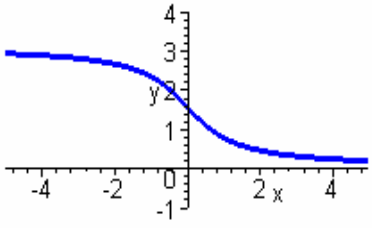
domain:
$-\infty < x \le -1, 1 \le x < \infty$
range:
$-\frac{\pi}{2} \le y < 0,\ 0 < y \le \frac{\pi}{2}$

$f(x) = arc\sec(x)$



domain:
$-\infty < x \le -1, 1 \le x < \infty$
range:
$0 \le y < \frac{\pi}{2},\ \frac{\pi}{2} < y \le \pi$

$f(x) = arc\cot(x)$



domain: $-\infty < x < \infty$
range: $0 < y < \pi$