A Modular Presentation System for the Calculus Sequence

5.6 The Logarithm Defined as an Integral

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The Natural Logarithmic Function

Definition

The **natural logarithmic function** is the function defined for $x > 0$ by

$$\ln x = \int_1^x \frac{1}{t} \, dt$$

The Derivative of ln

Note that

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

follows immediately from the definition of $\ln x$ and the Fundamental Theorem of Calculus.

It is also true that

$$\frac{d}{dx}[\ln |x|] = \frac{1}{x}$$

Application of the Derivative

EXAMPLE: Find $\frac{d}{dx}\left[\ln(2 + \cos x)\right]$

EXAMPLE: Find $\frac{d}{dx}\left[\ln\left(\cos^2 x\right)\right]$

EXAMPLE: Find $\displaystyle\int \frac{1}{x + 3}\, dx$

EXAMPLE: Find $\displaystyle\int \frac{x}{x^2 + 3}\, dx$

Properties of ln

Theorem

If a and b are positive numbers and r is any rational number, then

1. $\ln 1 = 0$

2. $\ln ab = \ln a + \ln b$

3. $\ln \frac{a}{b} = \ln a - \ln b$

4. $\ln a^r = r \ln a$

Asymptotes

EXAMPLE: Find the horizontal and vertical asymptotes of $y = \ln x$.

The Number e Redefined

Definition

$$e \text{ is the number such that } \ln e = 1$$