# A Modular Presentation System for the Calculus Sequence

## *5.5 The Substitution Rule*

Yaw Chang

Michael Freeze

Mathematics and Statistics UNC-Wilmington

# The Substitution Rule

## The Substitution Rule

If $u = g(x)$ is a differentiable function whose range is an interval $I$ and $f$ is continuous on $I$, then

$$\int f(g(x))g'(x)\,dx = \int f(u)du$$

# Applying the Substitution Rule

EXAMPLE: Find $\int e^{7x}\,dx$

EXAMPLE: Find $\int \tan x\,dx$

# Generalized Power Rule

## Generalized Power Rule for Integration

For $n \neq 1$,

$$\int [g(x)]^n \, g'(x) \, dx = \frac{[g(x)]^{n+1}}{n+1} + C$$

# Applying the Power Rule for Integration

EXAMPLE: Find $\int 3x^2\sqrt{x^3+2}\,dx$

EXAMPLE: Find $\int x(x^2+1)^4\,dx$

# Definite Substitution

## The Substitution Rule for Definite Integrals

If $g'$ is continuous on $[a, b]$ and $f$ is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

# Evaluating Definite Integrals

EXAMPLE: Find $\displaystyle\int_0^{\pi/6} \sin(2x)\cos^3(2x)\,dx$

EXAMPLE: Find $\displaystyle\int_2^5 x\sqrt{x^2-4}\,dx$

# Completing the Square

EXAMPLE: Find $\displaystyle\int \frac{7}{x^2 - 6x + 25}\, dx$

# Long Division

EXAMPLE: Find $\int \frac{x^2 - x}{x + 1}\, dx$

# Integrals of Symmetric Functions

## Integrals of Symmetric Functions

Suppose $f$ is continuous on $[-a, a]$.

(a) If $f$ is even, then

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

(b) If $f$ is odd, then

$$\int_{-a}^{a} f(x)\,dx = 0$$