A Modular Presentation System for the Calculus Sequence

5.3 The Fundamental Theorem of Calculus

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Functions Defined by Definite Integrals

- Functions Defined by Definite Integrals
- The Fundamental Theorem of Calculus, Part 1
- Example
- Example – cont.
- Example – cont.
- The Fundamental Theorem of Calculus, Part 2
- Examples

Let $f(t)$ be a continuous function on $[a, b]$.

Define $g(x)$ as

$$g(x) = \int_a^x f(t)dt \qquad x \in [a, b]$$

Example: Consider $f(t) = t$ and $g(x) = \int_0^x t dt$. Find $g(0)$, $g(1)$, and $g(2)$. Can you write an algebraic formula for $g(x)$?

The Fundamental Theorem of Calculus, Part 1

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If $f(t)$ is a continuous function on $[a, b]$, then the function $g(x)$ defined by

$$g(x) = \int_a^x f(t)dt \qquad x \in [a, b]$$

is a continuous function on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

Example

- Functions Defined by Definite Integrals
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- Example – cont.
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Example If

$$g(x) = \int_0^x \sqrt{1+t^2}\,dt,$$

then

$$g'(x) = \sqrt{1+x^2}.$$

Example – cont.

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Example Fresnel function:

$$S(x) = \int_0^x \sin(\frac{\pi t^2}{2})dt$$

then

$$S'(x) = \sin(\frac{\pi x^2}{2})$$

Let's take a look of these two function graphs.

Example – cont.

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Example (very important) Find the derivative of $g(x) = \int_1^{x^2} \sec t dt$.

$$\frac{d}{dx}\int_1^{x^2} \sec t dt = \frac{d}{dx}\int_1^{u} \sec t dt \quad (u = x^2)$$

$$= \frac{d}{du}\left(\int_1^{u} \sec t dt\right)\frac{du}{dx} \quad \text{(The Chain-Rule)}$$

$$= \sec u \frac{du}{dx} = \sec(x^2) \cdot 2x$$

The Fundamental Theorem of Calculus, Part 2

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If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, $F' = f$.

Examples

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Example Evaluate $\int_1^3 e^x dx$.

Example Evaluate $\int_3^6 \ln x dx$.

Example What is wrong?

$$\int_{-1}^{3} \frac{1}{x^2} dx = \frac{-1}{x}\Big|_{-1}^{3} = -\frac{-1}{3} - 1 = -\frac{4}{3}$$