A Modular Presentation System for the Calculus Sequence

5.1 Areas and Distances

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The Area Problem

PROBLEM Let $f(x) \geq 0$ be a continuous function on a closed interval$[a, b]$. How can we find the area under the curve $y = f(x)$ and above $x-$axis on $[a, b]$?

EXAMPLE Find the area under the parabola $y = x^2$ from 0 to 4.
Idea?

Sigma Notation

$\sum$ is known as the **sigma notation.**

$$\sum_{i=1}^{100} i = 1 + 2 + \cdots + 100$$

$$\sum_{i=1}^{10} i^2 = 1^2 + 2^2 + \cdots + 10^2$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2$$

Sums of Consecutive Integers

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n \tag{1}$$

$$= \frac{n(n+1)}{2}$$

Finding the Area of a Triangle Via Calculus

Sums of Consecutive Squares

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 \qquad (2)$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Finding the Area under a Parabola