



A Modular Presentation System for the Calculus Sequence

5.1 Areas and Distances

Yaw Chang

Michael Freeze

Mathematics and Statistics UNC-Wilmington



The Area Problem

▣ The Area Problem

- ▣ Sigma Notation
- ▣ Sums of Consecutive Integers
- ▣ Finding the Area of a Triangle Via Calculus
- ▣ Sums of Consecutive Squares
- ▣ Finding the Area under a Parabola

PROBLEM Let $f(x) \geq 0$ be a continuous function on a closed interval $[a, b]$. How can we find the area under the curve $y = f(x)$ and above x -axis on $[a, b]$?

EXAMPLE Find the area under the parabola $y = x^2$ from 0 to 4.
Idea?



Sigma Notation

▢ The Area Problem

▢ Sigma Notation

▢ Sums of Consecutive

Integers

▢ Finding the Area of a

Triangle Via Calculus

▢ Sums of Consecutive

Squares

▢ Finding the Area under a

Parabola

Σ is known as the **sigma notation**.

$$\sum_{i=1}^{100} i = 1 + 2 + \cdots + 100$$

$$\sum_{i=1}^{10} i^2 = 1^2 + 2^2 + \cdots + 10^2$$

$$\sum_{i=1}^n i^2 = 1^2 + 2^2 + \cdots + n^2$$



Sums of Consecutive Integers

- ▢ The Area Problem
- ▢ Sigma Notation
- ▢ Sums of Consecutive Integers
- ▢ Finding the Area of a Triangle Via Calculus
- ▢ Sums of Consecutive Squares
- ▢ Finding the Area under a Parabola

$$\begin{aligned}\sum_{i=1}^n i &= 1 + 2 + \cdots + n \\ &= \frac{n(n+1)}{2}\end{aligned}\tag{1}$$



Finding the Area of a Triangle Via Calculus

- ▢ The Area Problem
- ▢ Sigma Notation
- ▢ Sums of Consecutive Integers
- ▢ Finding the Area of a Triangle Via Calculus
- ▢ Sums of Consecutive Squares
- ▢ Finding the Area under a Parabola



Sums of Consecutive Squares

- ▢ The Area Problem
- ▢ Sigma Notation
- ▢ Sums of Consecutive Integers
- ▢ Finding the Area of a Triangle Via Calculus
- ▢ Sums of Consecutive Squares
- ▢ Finding the Area under a Parabola

$$\begin{aligned}\sum_{i=1}^n i^2 &= 1^2 + 2^2 + \cdots + n^2 \\ &= \frac{n(n+1)(2n+1)}{6}\end{aligned}\tag{2}$$



Finding the Area under a Parabola

- ◻ The Area Problem
- ◻ Sigma Notation
- ◻ Sums of Consecutive Integers
- ◻ Finding the Area of a Triangle Via Calculus
- ◻ Sums of Consecutive Squares
- ◻ Finding the Area under a Parabola