A Modular Presentation System for the Calculus Sequence

4.7 Optimization Problems

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Locating Absolute Extreme Values

First Derivative Test for Absolute Extreme Values

Suppose that c is a critical number of a continuous function f defined on an interval.

(a) If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f.

(b) If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f.

Strategy for Solving Optimization Problems

1. *Understand the Problem*
2. *Develop a Model of the Problem*
3. *Find the Domain of the Function*
4. *Identify the Critical Numbers and Endpoints*
5. *Solve the Model*
6. *Interpret the Solution*

Area Maximization

Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.

Inscribed Rectangle

Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

Poster Design

You are designing a rectangular poster to contain 50 in.^2 of printing with a 4-in. margin at the top and bottom and a 2-in. margin at each side. What overall dimensions will minimize the amount of paper used?

Airplane Landing Path

An airplane is flying at altitude H when it begins its descent to an airport runway that is at horizontal ground distance L from the airplane. Assume that the landing path of the airplane is the graph of a cubic polynomial function $y = ax^3 + bx^2 + cx + d$, where $y(0) = H$ and $y(L) = 0$. Determine a, b, c, d.

Physical Example

The illumination of an object by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources, one three times as strong as the other, are placed 10 ft apart, where should an object be placed on the line between the sources so as to receive the least illumination?