A Modular Presentation System for the Calculus Sequence

4.4 Indeterminate Forms and L'Hospital's Rule

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Indeterminate Form $0/0$

If the continuous functions $f(x)$ and $g(x)$ are both zero at $x = a$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

cannot be found by direct substitution. We develop a general method for evaluating such limits.

Weak L'Hospital's Rule

Theorem

Suppose that $f(a) = g(a) = 0$, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

Strong L'Hospital's Rule

Theorem

Suppose that $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$, that f and g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

provided that the limit on the right side exists.

Applying L'Hospital's Rule

EXAMPLE: Find $\lim_{x \to 0} \dfrac{5x - \sin x}{x}$

EXAMPLE: Find $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

EXAMPLE: Find $\lim_{x \to 0^+} \dfrac{\tan x}{x - \sin x}$

Other Indeterminate Forms I

EXAMPLE: ∞/∞ Find $\lim_{x\to\infty} \dfrac{x^2}{e^x}$

EXAMPLE: $\infty \cdot 0$ Find $\lim_{x\to\infty} x \sin\left(\dfrac{1}{x}\right)$

EXAMPLE: $\infty - \infty$ Find $\lim_{x\to 0^+} \left(\dfrac{1}{\sin x} - \dfrac{1}{x}\right)$

Other Indeterminate Forms II

EXAMPLE: 1^{∞} Find $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$

EXAMPLE: 0^{0} Find $\lim_{x\to 0^{+}} x^{x}$

EXAMPLE: ∞^{0} Find $\lim_{x\to 0^{+}}\left[\ln\left(\frac{1}{x}\right)\right]^{x}$