# A Modular Presentation System for the Calculus Sequence

## *4.3 How Derivatives Affect the Shape of a Graph*

Yaw Chang
Michael Freeze
Mathematics and Statistics UNC-Wilmington

# Increasing/Decreasing Test

**Increasing/Decreasing Test**

(a) If $f'(x) > 0$ on an interval, then $f$ is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then $f$ is decreasing on that interval.

# Applying the Inc/Dec Test

EXAMPLE: Find the intervals on which

$$y = x^2 - x - 1$$

is increasing/decreasing.

# First Derivative Test for Local Extrema

**The First Derivative Test**

Suppose that $c$ is a critical number of a continuous function $f$.

(a) If $f'$ changes from positive to negative at $c$, then $f$ has a local maximum at $c$.

(b) If $f'$ changes from negative to positive at $c$, then $f$ has a local minimum at $c$.

(c) If $f'$ does not change sign at $c$, then $f$ has no local maximum or minimum at $c$.

# Locating Local Extrema

EXAMPLE: Locate and identify the local extreme values of

$$y = 2x^4 - 4x^2 + 1$$

# Concavity

**Definition**

If the graph of $f$ lies above all of its tangents on an interval $I$, then it is called **concave upward** on $I$. If the graph of $f$ lies below all of its tangents on $I$, it is called **concave downward** on $I$.

# Concavity Test

**Concavity Test**

(a) If $f''(x) > 0$ for all $x$ in $I$, then the graph of $f$ is concave upward on $I$.

(b) If $f''(x) < 0$ for all $x$ in $I$, then the graph of $f$ is concave downward on $I$.

# Applying the Concavity Test

EXAMPLE: Find the intervals on which

$$y = -2x^3 + 6x^2 - 3$$

is concave upward/downward.

# Points of Inflection

**Definition**

A point $P$ on a curve $y = f(x)$ is called an **inflection point** if $f$ is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at $P$.

# Locating Inflection Points

EXAMPLE: Locate the inflection points of

$$y = -x^4 + 4x^3 - 4x + 1$$

# Second Derivative Test

**The Second Derivative Test**

Suppose $f''$ is continuous near $c$.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then $f$ has a local minimum at $c$.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then $f$ has a local maximum at $c$.