A Modular Presentation System for the Calculus Sequence

4.2 The Mean Value Theorem

Yaw Chang

Michael Freeze

Mathematics and Statistics UNC-Wilmington

Rolle's Theorem

Rolle's Theorem

Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).
3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

Mean Value Theorem

The Mean Value Theorem

Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$.

2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a)$$

Consequence 1

Functions with Zero Derivatives are Constant

Theorem

If $f'(x) = 0$ for all x in an open interval (a, b), then f is constant on (a, b).

Consequence 2

Functions with the Same Derivative Differ by a Constant

Theorem

If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + c$ where c is a constant.

Prove Identity

EXAMPLE: Probe the identity

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

Counting Roots

EXAMPLE: Assume that f is continuous on $[a, b]$ and differentiable on (a, b). Also assume that $f(a)$ and $f(b)$ have opposite signs and that $f' \neq 0$ between a and b. Show that $f(x) = 0$ exactly once between a and b.