A Modular Presentation System for the Calculus Sequence

4.1 Maximum and Minimum Values

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The Drilling-Rig Problem

A drilling rig 12 mi offshore is to be connected by pipe to a refinery onshore, 20 mi straight down the coast from the rig. If underwater pipe costs $50,000 per mile and land-based pipe costs $30,000 per mile, what combination of the two will give the least expensive connection?

Absolute Extreme Values

Definition

Let f be a function with domain D. Then $f(c)$ is the

(a) **absolute maximum value** on D if and only if $f(x) \leq f(c)$ for all x in D

(b) **absolute minimum value** on D if and only if $f(x) \geq f(c)$ for all x in D.

Investigating Extreme Values

EXAMPLE: Determine the absolute maximum and minimum values of $\cos x$ on the interval $[-\pi/2, \pi/2]$.

The Extreme Value Theorem

The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Local Extreme Values

Definition

Let c be an interior point of the domain of the function f. Then $f(c)$ is a

(a) **local maximum value** at c if and only if $f(x) \leq f(c)$ for all x in some open interval containing c.

(b) **local minimum value** at c if and only if $f(x) \geq f(c)$ for all x in some open interval containing c.

Locating Extreme Values

Fermat's Theorem

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Critical Number

Definition

A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Finding Extreme Values

How to Find the Absolute Extrema of a Continuous Function on a Closed Interval

1. Evaluate f at all critical numbers and endpoints.
2. Take the largest and smallest of these values.

Solution to the Drilling-Rig Problem

The least expensive connection costs $1,080,000 and it is achieved by running the line underwater to the point on shore 11 miles from the refinery.