A Modular Presentation System for the Calculus Sequence

3.8 Derivatives of Logarithmic Functions

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Derivatives of Log Functions

$$\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}}$$

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

Chain Rule Form

$$\boxed{\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}}$$

$$\boxed{\frac{d}{dx}\left[\ln g(x)\right] = \frac{g'(x)}{g(x)}}$$

Applying the Rules

EXAMPLE: Find

$$\frac{d}{dx}[\ln(\cos x)]$$

EXAMPLE: Find

$$\frac{d}{dx}\left[\frac{\ln x}{1+\ln x}\right]$$

Laws of Logarithms

For any expressions $u > 0$ and $v > 0$,

1. $\ln(uv) = \ln(u) + \ln(v)$
2. $\ln\left(\frac{u}{v}\right) = \ln(u) - \ln(v)$
3. $\ln(u^n) = n\ln(u)$

Logarithmic Differentiation

Since the Laws of Logarithms allow us essentially to rewrite products as sums and quotients as differences, we may use logarithms to expedite the process of differentiating complicated products and quotients.

The Three Steps of Logarithmic Differentiation

Let $f(x)$ be a given function.

1. Let $g(x) = \ln(f(x))$.

2. Differentiate both side with respect to x.

3. Solve the resulting equation for $g'(x)$.

Applying Logarithmic Differentiation

EXAMPLE: Differentiate

$$f(x) = \frac{(x^2+1)(x+3)^{1/2}}{x-1}$$

EXAMPLE: Differentiate

$$f(x) = \sqrt[3]{\frac{x(x-2)}{x^2+1}}$$

The Number e as a Limit

$$\boxed{e = \lim_{x \to 0}(1+x)^{1/x}}$$

$$\boxed{e = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^{n}}$$