A Modular Presentation System for the Calculus Sequence

3.6 Implicit Differentiation

Yaw Chang

Michael Freeze

Mathematics and Statistics UNC-Wilmington

Questions:

Consider the following questions:

1. Find an equation of the tangent line to the curve $y^2 = x^2 e^x$ at $(0, 0)$.
2. Find an equation of the tangent line to the curve $x^2 + y^2 = 25$ at $(\pm 3, \pm 4)$.
3. Find an equation of the tangent line to the curve $x^3 + y^3 = 6xy$ at $(3, 3)$.
4. Find an equation of the tangent line to the curve $\sin(x + y) = y^2 \cos(x)$ at $(0, 0)$.

Implicitly Defined Functions

EXAMPLE: The standard equation for the circle of radius 2 centered at the origin,

$$x^2 + y^2 = 4$$

is satisfied for many functions $y = f(x)$.

We would like to be able to find the slope at various points on such a curve without explicitly finding a formula in x first.

The Four Steps of Implicit Differentiation

1. Differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.
2. Collect the terms with dy/dx on one side of the equation.
3. Factor out dy/dx.
4. Solve for dy/dx.

Differentiating Implicitly

EXAMPLE: Find dy/dx for the curve defined by

$$y^2 = x$$

EXAMPLE: Find dy/dx for the Folium of Descartes given by

$$x^3 + y^3 - 9xy = 0$$

Derivatives of Inverse Functions

Note that if $y = f^{-1}(x)$, then

$$\begin{aligned} f(y) &= x \\ f'(y) \cdot y' &= 1 \\ y' &= \frac{1}{f'(y)} \end{aligned}$$

This approach allows us to differentiate inverses of elementary functions.

Derivatives of Some Inverse Trig Functions

EXAMPLE: Find

$$\frac{d}{dx}\left(\sin^{-1} x\right)$$

EXAMPLE: Find

$$\frac{d}{dx}\left(\tan^{-1} x\right)$$

Derivatives of Inverse Trig Functions

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$