

A Modular Presentation System for the Calculus Sequence

3.5 The Chain Rule

Yaw Chang Michael Freeze

Mathematics and Statistics UNC-Wilmington



C The Chain Rule

- The Chain Rule in Leibniz Notation
- C Inner and Outer Functions
- Applying the Chain Rule
- Power Chain Rule
- Applying the Power Chain Rule
- ${f o}$ Derivative of a^x
- OPartial Proof of Chain Rule
- Repeated Use of the Chain Rule

The Chain Rule

If *f* and *g* are differentiable and $F = f \circ g$ is the composite function defined by F(x) = f(g(x)), then *F* is differentiable and *F'* is given by the product

$$F'(x) = f'(g(x))g'(x)$$



The Chain Rule in Leibniz Notation

O The Chain Rule

- The Chain Rule in Leibniz Notation
- Inner and Outer Functions
- Applying the Chain Rule
- Power Chain Rule
- Applying the Power Chain Rule
- ${f o}$ Derivative of a^x
- C Partial Proof of Chain Rule
- Repeated Use of the Chain Rule

The Chain Rule (Leibniz Notation) If y = f(u) and u = g(x) are both differentiable functions, then du = du du

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$



Inner and Outer Functions

The Chain Rule

• The Chain Rule in Leibniz

C Inner and Outer Functions

• Applying the Chain Rule

O Power Chain Rule

• Applying the Power Chain Rule

 $old Derivative of a^x$

C Partial Proof of Chain Rule

• Repeated Use of the Chain Rule

EXAMPLE: Find inner and outer functions for

$$y = \sin(x^2 + x)$$

EXAMPLE: Find inner and outer functions for

$$y = e^{5x}$$



Applying the Chain Rule

The Chain Rule

• The Chain Rule in Leibniz Notation

Inner and Outer Functions

• Applying the Chain Rule

C Power Chain Rule

C Applying the Power Chain Rule

 ${\bf O}$ Derivative of $a^{\,x}$

OPartial Proof of Chain Rule

• Repeated Use of the Chain Rule

EXAMPLE: Find the derivative of

 $y = \sin(x^2 + x)$

EXAMPLE: Find the derivative of

$$y = e^{5x}$$



Power Chain Rule

The Chain Rule
The Chain Rule in Leibniz Notation

OInner and Outer Functions

Applying the Chain Rule

Power Chain Rule

• Applying the Power Chain Rule • Derivative of a^x

C Partial Proof of Chain Rule

• Repeated Use of the Chain Rule

The Power Chain Rule

If *n* is any real number and u = g(x) is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

Alternatively,

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$



Applying the Power Chain Rule

• The Chain Rule

C The Chain Rule in Leibniz

Notation Conner and Outer Functions

• Applying the Chain Rule

• Power Chain Rule

C Applying the Power Chain Rule

 $oldsymbol{\circ}$ Derivative of a^x

OPartial Proof of Chain Rule

• Repeated Use of the Chain Rule **EXAMPLE:** Find the derivative of

$$y = (x+5)^2$$

EXAMPLE: Find the derivative of

$$y = (x^2 + 5x + 7)^{10}$$



C The Chain Rule

C The Chain Rule in Leibniz

Notation

- OInner and Outer Functions
- O Applying the Chain Rule
- OPower Chain Rule
- Applying the Power Chain Rule

O Derivative of a^x

• Partial Proof of Chain Rule • Repeated Use of the Chain Rule

EXAMPLE: Noting that

$$a^x = e^{(x \ln a)}$$

find the derivative of

$$y = a^x$$



Partial Proof of Chain Rule

 $\frac{dy}{dx}$

| C The Chain Rule | |
|-----------------------------|--|
| C The Chain Rule in Leibniz | |
| Notation | |
| Inner and Outer Functions | |
| Applying the Chain Rule | |
| Power Chain Rule | |
| Applying the Power Chain | |
| Rule | |
| ODerivative of a^x | |
| Partial Proof of Chain Rule | |
| Repeated Use of the Chain | |

Rule

 $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ Δy Δu lim $\Delta x \rightarrow 0 \Delta u$ Δx $\frac{\Delta y}{2}$. Δu lim lim $\lim_{\Delta x \to 0} \frac{1}{\Delta u} \cdot$ $\Delta x \rightarrow 0 \Delta x$ $\lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}$ dy du $\overline{du} \, \overline{dx}$

Where is the flaw in this argument?



Repeated Use of the Chain Rule

The Chain Rule
The Chain Rule in Leibniz Notation
Inner and Outer Functions
Applying the Chain Rule
Power Chain Rule
Applying the Power Chain Rule
Derivative of a ^x
Partial Proof of Chain Rule

 Repeated Use of the Chain Rule Where do I start? Consider the function evaluation ...

EXAMPLE: Find the derivative of

$$y = e^{\sin(5x)}$$

EXAMPLE: Find the derivative of

$$y = \sin^2(7x)$$