A Modular Presentation System for the Calculus Sequence

3.4 Derivatives of Trigonometric Functions

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Derivative of the Sine Function

Recalling that

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$

and

$$\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 0$$

we may use the limit definition of derivative to show that

$$\frac{d}{dx}(\sin x) = \cos x$$

Derivatives Involving Sine

EXAMPLE: Find the derivative of

$$y = x^2 - \sin x$$

EXAMPLE: Find the derivative of

$$y = \frac{\sin x}{x}$$

Derivative of the Cosine Function

$$\frac{d}{dx}(\cos x) = -\sin x$$

Derivatives Involving Sine and Cosine

EXAMPLE: Find the derivative of

$$y = \sin x \cos x$$

EXAMPLE: Find the derivative of

$$y = \frac{\cos x}{1 - \sin x}$$

Simple Harmonic Motion

EXAMPLE: A body hanging from a spring is stretched 5 units beyond its rest position and released at time $t = 0$ to bob up and down. Its position at any later time t is

$$s = 5 \cos t$$

What are its velocity and acceleration at time t?

Derivatives of Other Basic Trig Functions

Derivatives of Trigonometric Functions

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$

Limits

- Use $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ to find other limits.

Examples:

1.
$$\lim_{x\to 0} \frac{\sin(7x)}{4x}$$

2.
$$\lim_{x\to 0} \frac{\sin(\cos x)}{\sec x}$$

3.
$$\lim_{x\to 0} x \cot x$$