A Modular Presentation System for the Calculus Sequence

3.2 Derivatives of Products, Quotients, and Negative Powers

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Derivative Product Rule

The Product Rule

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Applying the Product Rule

EXAMPLE: Find the derivative of

$$y = \frac{1}{x}\left(x^2 + \frac{1}{x}\right)$$

EXAMPLE: Find the derivative of

$$y = xe^x$$

Derivative Quotient Rule

The Quotient Rule

If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Derivative Reciprocal Rule

The Reciprocal Rule

If g is differentiable, then

$$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{[g(x)]^2}$$

Choosing Which Rule to Use

EXAMPLE: Find the derivative of

$$y = \frac{(x-1)(x^2-2x)}{x^4}$$

A Witch of Agnesi

EXAMPLE: Find an equation for the tangent line to the *witch of Agnesi* given by

$$f(x) = \frac{8}{x^2 + 4}$$

at the point $(2, 1)$.

A Serpentine

EXAMPLE: Find an equation for the tangent line to *Newton's serpentine* given by

$$f(x) = \frac{4x}{x^2 + 1}$$

at the point $(1, 2)$.