A Modular Presentation System for the Calculus Sequence

2.8,2.9 Derivatives

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Definition of Derivative

Definition

The **derivative of a function** f **at a number** a, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

How to Apply the Definition

1. Write expressions for $f(x)$ and $f(x+h)$.
2. Expand and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$.
3. Using the simplified quotient, find $f'(x)$ by evaluating the limit $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

Applying the Definition

EXAMPLE: Find the derivative of $y = \sqrt{x}$ for $x > 0$.

EXAMPLE: Find the derivative of $y = 4 - x^2$.

Interpretation: Slope

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

Interpretation: Rate of Change

The derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$.

Other Notations

Symbol	Verbalization	Utility
y'	"y prime"	Nice and brief but does not name the independent variable
$\frac{dy}{dx}$	"$dy\ dx$"	Names the variables and uses d for derivative
$\frac{df}{dx}$	"$df\ dx$"	Emphasizes the function's name
$\frac{d}{dx}f(x)$	"ddx of $f(x)$"	Emphasizes the idea that differentiation is an operation performed on f

Differentiability implies Continuity

Theorem

If f is differentiable at a, then f is continuous at a.

WARNING: The converse of this theorem is false. There are functions that are continuous but not differentiable.

Types of Nondifferentiability

There are several basic situations where a function fails to be differntiable:

1. Corner $|x|$ at $x = 0$
2. Cusp $|x|^{1/3}$ at $x = 0$
3. Vertical Tangent $\sqrt[3]{x}$ at $x = 0$
4. Discontinuity $\frac{x^2-9}{x-3}$ at $x = 3$