

A Modular Presentation System for the Calculus Sequence

2.8,2.9 Derivatives

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Definition of Derivative

• Definition of Derivative

- How to Apply the Definition
- Applying the Definition
- Interpretation: Slope
- C Interpretation: Rate of Change
- Other Notations
- Differentiability implies Continuity
- C Intermediate Value Property
- Types of
- Nondifferentiability

Definition

The derivative of a function f at a number a, denoted by f'(a), is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.



How to Apply the Definition

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1. Write expressions for f(x) and f(x+h).

- 2. Expand and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$.
- 3. Using the simplified quotient, find f'(x) by evaluating the limit $f'(x) = \lim_{h \to 0} \frac{f(x+h) f(x)}{h}$.



Applying the Definition

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EXAMPLE: Find the derivative of $y = \sqrt{x}$ for x > 0.

EXAMPLE: Find the derivative of $y = 4 - x^2$.



Interpretation: Slope

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The tangent line to y = f(x) at (a, f(a)) is the line through (a, f(a)) whose slope is equal to f'(a), the derivative of f at a.



Interpretation: Rate of Change

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The derivative f'(a) is the instantaneous rate of change of y = f(x) with respect to x when x = a.



Other Notations

| Pefinition of Derivative How to Apply the Definition Applying the Definition Interpretation: Slope Interpretation: Rate of Change Other Notations Differentiability implies Continuity Intermediate Value Property Types of Nondifferentiability | Symbol y' | Verbalization "y prime" | Utility Nice and brief but does not name the independent variable |
|--|--------------------|----------------------------|---|
| | $rac{dy}{dx}$ | " $dy dx$ " | Names the variables and uses d for derivative |
| | $rac{df}{dx}$ | " $df dx$ " | Emphasizes the function's name |
| | $\frac{d}{dx}f(x)$ | " ddx of $f(x)$ " | Emphasizes the idea that differentiation is an operation performed on f |
| | | | |



Differentiability implies Continuity

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Theorem

If f is differentiable at a, then f is continuous at a.

WARNING: The converse of this theorem is false. There are functions that are continuous but not differentiable.



Types of Nondifferentiability

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There are several basic situations where a function fails to be differntiable:

1. Corner |x| at x = 0

2. Cusp $|x|^{1/3}$ at x = 0

3. Vertical Tangent $\sqrt[3]{x}$ at x = 0

4. Discontinuity $\frac{x^2-9}{x-3}$ at x=3