A Modular Presentation System for the Calculus Sequence

2.6 Limits at Infinity: Horizontal Asymptotes

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Informal Definition

Definition

Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

Horizontal Asymptotes

Definition

The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = L \quad \text{or} \quad \lim_{x \to -\infty} f(x) = L$$

A Special Limit

Theorem

If $r > 0$ is a rational number, then

$$\lim_{x\to\infty} \frac{1}{x^r} = 0$$

If $r > 0$ is a rational number such that x^r is defined for all x, then

$$\lim_{x\to-\infty} \frac{1}{x^r} = 0$$

Finding Horizontal Asymptotes

EXAMPLE: Find the horizontal asymptote(s) of

$$f(x) = \frac{2x^2 + x - 1}{3x^2 - 1}$$

Limits of Rational Functions

Limits of rational functions as $x \to \pm\infty$ are of three basic types:

1. Numerator and Denominator of Same Degree

2. Degree of Numerator Less Than Degree of Denominator

3. Degree of Numerator Greater Than Degree of Denominator

Examples

EXAMPLE: Find $\lim_{x\to\infty} \dfrac{5x^2+8x-3}{3x^2+2}$.

EXAMPLE: Find $\lim_{x\to-\infty} \dfrac{11x+2}{2x^3-1}$.

EXAMPLE: Find $\lim_{x\to\infty} \dfrac{2x^2-3}{7x+4}$.

Oblique Asymptotes

If the degree of the numerator is one greater than the degree of the denominator, the graph of the rational function $f(x)$ has an **oblique asymptote**.

EXAMPLE: Find the oblique asymptote of

$$\lim_{x\to\infty} \frac{2x^2-3}{7x+4}$$

End Behavior Models

Definition

The function g is

(a) a **right end behavior model** for f if and only if

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$$

(b) a **left end behavior model** for f if and only if

$$\lim_{x \to -\infty} \frac{f(x)}{g(x)} = 1$$

Example

EXAMPLE: Find end behavior models for

$$f(x) = x + e^{-x}$$