A Modular Presentation System for the Calculus Sequence

2.3 Calculating Limits Using the Limits Laws

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Two Special Limits

Suppose that c is a constant.

Constant Law

$$\lim_{x \to a} c = c$$

Identity Law

$$\lim_{x \to a} x = a$$

Limit Laws

Suppose that c is a constant and the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

Sum Law

$$\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

Constant Multiple Law

$$\lim_{x \to a}[cf(x)] = c \lim_{x \to a} f(x)$$

More Limit Laws

Difference Law

$$\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

Product Law

$$\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

Even More Limit Laws

Quotient Law

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} \quad \text{if } \lim_{x \to a} g(x) \neq 0$$

Power Law

$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n \quad \text{where } n \text{ is a positive integer}$$

Examples

EXAMPLE: Find $\lim_{x \to 2}(x^2 + 6x + 8)$.

EXAMPLE: Find $\lim_{x \to -3} \dfrac{x^2+1}{x}$.

Direct Substitution

Direct Substitution Property

If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a)$$

REMARK: Actually the above result is true for all functions if a is not on the boundary of the domain.

Example: $f(x) = \sqrt{x}$.

Eliminating Zero Denominators

EXAMPLE Canceling a Common Factor

Find $\displaystyle\lim_{x\to 1} \frac{x^2+x-2}{x^2-x}$.

EXAMPLE Creating and Canceling a Common Factor

Find $\displaystyle\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$.

The Squeeze Theorem

The Squeeze Theorem

If

$$f(x) \leq g(x) \leq h(x)$$

when x is near a (except possibly at $x = a$) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$

Examples

EXAMPLE: Show that $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.

EXAMPLE: Show that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$.