

A Modular Presentation System for the Calculus Sequence

2.3 Calculating Limits Using the Limits Laws

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C Two Special Limits

Limit Laws

• More Limit Laws

C Even More Limit Laws

• Examples

O Direct Substitution

C Eliminating Zero

Denominators

• The Squeeze Theorem

Examples

Suppose that c is a constant.

Constant Law

$$\lim_{x \to a} c = c$$

Identity Law $\lim_{x \to a} x = a$



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Suppose that c is a constant and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Sum Law

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

Constant Multiple Law

$$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$$



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Difference Law

$$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

Product Law

$$\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$



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Quotient Law

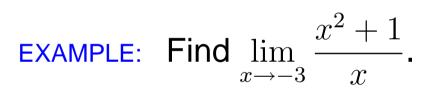
$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{if } \lim_{x \to a} g(x) \neq 0$$

Power Law $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n \text{ where } n \text{ is a positive integer}$



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EXAMPLE: Find $\lim_{x \to 2} (x^2 + 6x + 8)$.





Direct Substitution

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Denominators • The Squeeze Theorem • Examples

Direct Substitution Property

If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a)$$

REMARK: Actually the above result is true for all functions if *a* is not on the boundary of the domain.

Example: $f(x) = \sqrt{x}$.



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• The Squeeze Theorem • Examples EXAMPLE Canceling a Common Factor Find $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

EXAMPLE Creating and Canceling a Common Factor Find $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$.



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The Squeeze Theorem

 $f(x) \le g(x) \le h(x)$

when x is near a (except possibly at x = a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

lf

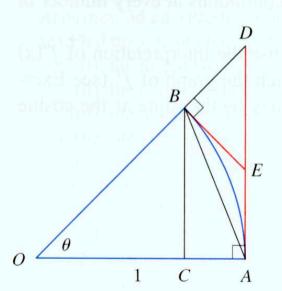
$$\lim_{x \to a} g(x) = L$$



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EXAMPLE: Show that
$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$$

EXAMPLE: Show that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1.$



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