A Modular Presentation System for the Calculus Sequence

Chapter 1: Functions

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1.6 Inverse Functions and Logarithms

- Definition: A function f is called a *one-to-one function* if

$$f(x_1) \neq f(x_2) \quad \text{whenever} \quad x_1 \neq x_2$$

- Horizontal Line Test: Determining whether a function is one-to-one geometrically.
- Definition: Let f be a one-to-one function with domain A and range B. Then its *inverse function* has domain B and range A and is defined by

$$f^{-1}(y) = x \Leftrightarrow f(x) = y \quad \text{for any} \quad y \in B.$$

- How to check? $f(x)$ and $g(x)$ form a pair of *inverse function* if and only if

$$(f \circ g)(x) = (g \circ f)(x) = x.$$

- Example: Let $f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$. Because

$$(f \circ g)(x) = f(g(x)) = (\sqrt[3]{x-2})^3 + 2 = x - 2 + 2 = x,$$

and

$$(g \circ f)(x) = g(f(x)) = \sqrt[3]{(x^3 + 2 - 2} = \sqrt[3]{x^3} = x,$$

so f and g form a pair of inverse functions.

- Important
  1. $$f^{-1}(x) \neq \frac{1}{f(x)}$$
  2. domain of f^{-1} = range of f
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- Finding the inverse function:
  1. Write $y = f(x)$.
  2. Interchange the variables x and y.
  3. Solve the variable y in terms of x.
- Example: Find the inverse function of $f(x) = \sqrt{x^2 + 2}$.

$$y = \sqrt{x^2 + 2}$$

$$x = \sqrt{y^2 + 2}$$

$$x^2 = y^2 + 2$$

$$y^2 = x^2 - 2$$

$$y = \sqrt{x^2 - 2} \quad \text{or} \quad f^{-1}(x) = \sqrt{x^2 - 2}$$

Logarithmic Functions

- Definition: Let $a > 0$, $a \neq 1$ and $x > 0$. Then

$$\log_a x = y \Leftrightarrow a^y = x$$

- Logarithmic function: $f(x) = \log_a(x)$
- Properties:
  1. The domain of the function: $x > 0$.
  2. When $a > 1$, $f(x)$ is increasing.
  3. When $0 < a < 1$, $f(x)$ is decreasing.
  4. Is there any $x-$intercept of the function graph?
  5. Is there any $y-$intercept of the function graph?

Laws of Logarithms:

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a(x/y) = \log_a x - \log_a y$
3. $\log_a(x^r) = r \log_a x$
4. $\log_a 1 = 0$

Natural Logarithms:

$$\log_e x = \ln x$$

Change of Base Formula:

$$\log_a x = \frac{\ln x}{\ln a} = \frac{\log x}{\log a}$$

Solving equations:

$$\ln x + \ln(x-2) = \ln(x+4)$$

$$3^{1-2x} = 4^x$$