A Modular Presentation System for the Calculus Sequence

Chapter 1: Functions

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1.3 New Functions from Old Functions

- Transformation of functions: Let $f(x)$ be a given function and $c > 0$ be a constant. To obtain the graph of
  1. $y = f(x) + c$, shift the graph of $y = f(x)$ c units upward. (Compare the graph of $y = |x|$ and $y = |x| + 2$.)
  2. $y = f(x) - c$, shift the graph of $y = f(x)$ c units downward. (Compare the graph of $y = |x|$ and $y = |x| - 2$.)
  3. $y = f(x - c)$, shift the graph of $y = f(x)$ c units to the right. (Compare the graph of $y = |x|$ and $y = |x - 2|$.)
  4. $y = f(x + c)$, shift the graph of $y = f(x)$ c units to the left. (Compare the graph of $y = |x|$ and $y = |x + 2|$.)

- Vertical and Horizontal Stretching and Reflecting: Let $f(x)$ be a given function and $c > 1$ be a constant. To obtain the graph of
  1. $y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c. (Compare $y = \sin(x)$ and $y = 2\sin(x)$.)
  2. $y = (1/c)f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c. (Compare $y = \sin(x)$ and $y = \frac{1}{2}\sin(x)$.)
  3. $y = f(cx)$, compress the graph of $y = f(x)$ horizontally by a factor of c. (Compare $y = \sin(x)$ and $y = \sin(2x)$.)
  4. $y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c. (Compare $y = \sin(x)$ and $y = \sin(x/2)$.)
  5. $y = -f(x)$, reflect the graph of $y = f(x)$ about the $x-$axis. (Compare $y = |x|$ and $y = -|x|$.)
  6. $y = f(-x)$, reflect the graph of $y = f(x)$ about the $y-$axis. (Compare $y = x^3$ and $y = (-x)^3 = -x^3$.)

- Algebra of Functions: Let f and g be two functions with domains A and B. Then
  1. $(f+g)(x) = f(x) + g(x)$ and domain = $A \bigcap B$
  2. $(f-g)(x) = f(x) - g(x)$ and domain = $A \bigcap B$
  3. $(fg)(x) = f(x)g(x)$ and domain = $A \bigcap B$
  4. $(f/g)(x) = f(x)/g(x)$ and domain = $\{x \in A \bigcap B | g(x) \neq 0\}$

  (Consider $f(x) = x^2$ and $g(x) = \sqrt{x}$.)
- Composition: Let f and g be two functions, the *composite* function $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x)).$$

1.5 Exponential Functions

- Definition: An *exponential function* is a function of the form

$$f(x) = a^x$$

  where $a > 0, a \neq 1$ is a constant.

- Properties:
  1. $f(x) > 0$ for all x.
  2. When $a > 1$, $f(x)$ is increasing.
  3. When $0 < a < 1$, $f(x)$ is decreasing.
  4. Is there any $x-$intercept of the function graph?
  5. Is there any $y-$intercept of the function graph?

Laws of Exponents

1. $a^{x+y} = a^x a^y$
2. $a^{x-y} = a^x / a^y$
3. $(a^x)^y = a^{xy}$
4. $(ab)^x = a^x b^x$
5. $a^0 = 1$

Solving equations:

$$3^{x+1} = 81$$

$$e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$$