A Modular Presentation System for the Calculus Sequence

Chapter 1: Functions

Yaw Chang

Michael Freeze

Mathematics and Statistics UNC-Wilmington

1.1 Four Ways to Represent a Function

- What is a function?
  Let A and B be two sets. A *function* f from A to B is a *relation* between A and B such that any element x in A, there is a *unique* corresponding element y in B.
- Representation of functions
  - verbally (by a description in words)
  - numerically (by a table)
  - visually (by a graph)
  - algebraically (by an explicit formula)

- Domain of a function
  - defined strictly from algebra sense
  - defined according to application
- The Vertical Line Test
- Piecewise functions
- Even and Odd functions
- Increasing and Decreasing Functions:
  A function f is called *increasing* on an interval I if

$$f(x) < f(y)$$

  whenever $x < y$ in I.
  It is called *decreasing* on I if

$$f(x) > f(y)$$

  whenever $x < y$ in I.

1.2 Some Basic Functions

- Polynomials

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where a_n, a_{n-1}, ..., a_0 are constants called the *coefficients* of the polynomial.
Domain?

- Power Functions

$$f(x) = x^a$$

where a is a constant. (review the conversion between rational exponents and radical sign.)

Basic Functions (cont.)

- Rational Functions

$$f(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials.
Domain: $\{x|Q(x) \neq 0\}$.

- Algebraic Functions: Those function can be constructed from polynomials.

$$f(x) = \sqrt{x^2+1}, \quad g(x) = x^3 - \frac{x^3 - x - 1}{\sqrt[3]{x-1}}$$

- Trigonometric Functions

$$\sin(x), \quad \cos(x), \quad \tan(x), \quad \cot(x), \quad \sec(x), \quad \csc(x)$$

Basic Functions (cont.)

- Exponential Functions

$$f(x) = a^x$$

where $a > 0, a \neq 1$. In particular $f(x) = e^x$.

- Logarithmic Functions

$$f(x) = \log_a x$$

where $a > 0, a \neq 1$ and $x > 0$.
In particular, $f(x) = \ln x = \log_e x$.

- Transcentdental Functions: Those functions are not algebraic.
  Examples: Trigonometric, inverse trigonometric, exponential and logarithmic functions.