Networks II: Flows

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Network Environ Analysis

Path Analysis – a_{ij} – enumerates number of pathways in a network

- **Flow Analysis** ($g_{ij} = f_{ij}/T_j$) – identifies flow intensities along indirect pathways
- **Storage Analysis** ($c_{ij} = f_{ij}/x_j$) – identifies storage intensities along indirect pathways
- **Utility Analysis** ($d_{ij} = (f_{ij} – f_{ji})/T_i$) – identifies utility intensities along indirect pathways

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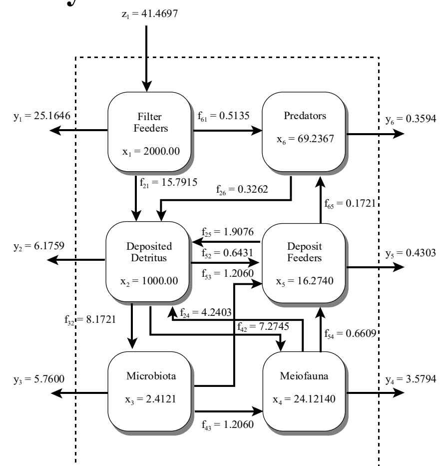
Oyster Reef Model



$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Dame and Patten 1981 – flow is in kcal/(day m^2), storage in kcal/m^2

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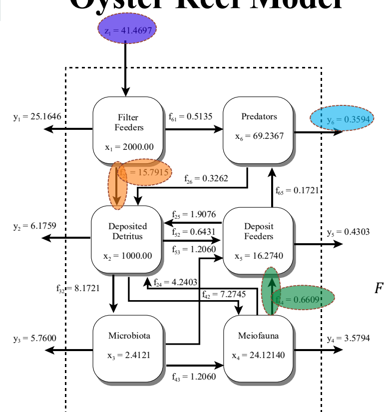
Oyster Reef Model



$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Intercompartmental flows (F) — External inputs (z)

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 15.79 & 0 & 0 & 4.24 & 1.91 & 0.32 \\ 0 & 8.17 & 0 & 0 & 0 & 0 \\ 0 & 7.27 & 1.21 & 0 & 0 & 0 \\ 0 & 0.64 & 1.21 & 0.66 & 0 & 0 \\ 0.51 & 0 & 0 & 0 & 0.17 & 0 \end{bmatrix} \quad \begin{bmatrix} 41.4697 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[25.16 \quad 6.18 \quad 5.76 \quad 3.58 \quad 0.43 \quad 0.36]$$

External outputs (y)

Dame and Patten 1981 – flow is in kcal/(day m^2), storage in kcal/m^2

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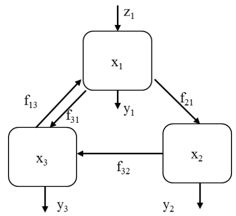
Functional Analysis (weighted digraphs)

Conservative flows along arrows (oriented from column j to row i -- f_{ij} is flow from j to i. f_{21} is the flow from X_1 to X_2)



Throughflow at node i:

$$T_{1,in} = z_1 + f_{13} \qquad T_{1,out} = f_{21} + f_{31} + y_1$$

$$T_{2,in} = f_{21} \qquad T_{2,out} = f_{32} + y_2$$

$$T_{3,in} = f_{32} + f_{31} \qquad T_{3,out} = f_{13} + y_3$$

$$T_{i,in} = \sum_{j=1}^{n} f_{ij} + z_i \qquad T_{i,out} = \sum_{j=1}^{n} f_{ji} + y_i$$

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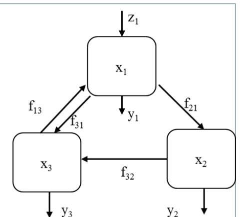
at steady state: $T_{i,in} = T_{i,out} = T_i$



Total System Throughflow (TST): $TST = \sum_{i=1}^{n} T_i$

$$TST = T_{1,in} + T_{2\ in} + T_{3\ in}$$

$$TST = T_{1,out} + T_{2,out} + T_{3\ out}$$

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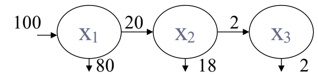
Flow Analysis



Adjacency matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Inter-compartmental flows

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 20 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

inputs

$$z = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

outputs

$$y = \begin{bmatrix} 80 & 18 & 2 \end{bmatrix}$$

Total flow through each compartment $T = \begin{bmatrix} 100 \\ 20 \\ 2 \end{bmatrix}$

The outflow (time forward, input driven) fractions are given by g_{ij} where

$$g_{ij} = \frac{f_{ij}}{T_j} \qquad G = \begin{bmatrix} 0 & 0 & 0 \\ {}^{20}/_{100} & 0 & 0 \\ 0 & {}^{2}/_{20} & 0 \end{bmatrix} \qquad G = \begin{bmatrix} 0 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}$$

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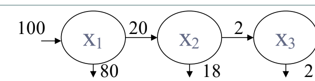

The outflow (time forward, input driven) fractions are given by g_{ij} where

$$g_{ij} = \frac{f_{ij}}{T_j} \qquad G = \begin{bmatrix} 0 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}$$

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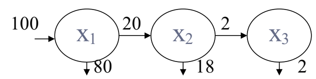
Just as powers of A gave higher order pathways,
Powers of G give flow transfers along higher order pathways.

G^2 gives the fraction of flow leaving j that took 2 steps to reach i.

$$G^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.02 & 0 & 0 \end{bmatrix}$$



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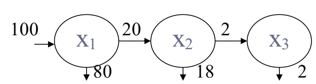
Continuing:

G^3 gives the fraction of flow leaving j that took 3 steps to reach i.

$$G^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



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Summarizing:

G^2 gives transfers over pathways of length 2
G^3 gives transfers over pathways of length 3, etc., i.e.,

G^m gives transfers over pathways of length m

Summing over m=1→∞ gives powers over all pathways

$\sum_{m=0}^{\infty} G^m$ where $\sum_{m=2}^{\infty} G^m$ represent indirect transfers

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Unlike like powers of A, powers of G get smaller and the series converges

$$N = \sum_{m=0}^{\infty} G^m \equiv (I - G)^{-1}$$

N is the INTEGRAL output flow matrix since it includes direct and all indirect flows

N maps input into throughflows along all pathways

$$T = Nz$$

$$T = \left(\sum_{m=0}^{\infty} G^m \right) z \equiv (I - G)^{-1} z$$

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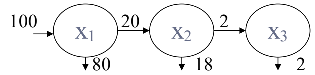
Concluding:



Integral matrix N gives the flow leaving j that took all steps to reach i.

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ .002 & 0.1 & 1 \end{bmatrix}$$

$$T = Nz$$

$$Nz = \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ .002 & 0.1 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \qquad T = \begin{bmatrix} 100 \\ 20 \\ 2 \end{bmatrix}$$

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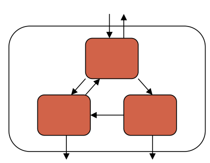
Propagation of network indirect effects

Flow: $\mathbf{N} = \mathbf{I} + \mathbf{G} + \mathbf{G}^2 + \mathbf{G}^3 + \mathbf{G}^4 + \ldots$

integral = *initial input* + *direct* + *indirect*



Key findings:

- Quantify input and output flow
- Indirect flows > direct flows
- Flows are well mixed
- Mutualistic relations dominate

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Decomposition of the power series explicitly shows the contribution due to indirect pathways

$$\underset{\text{integral}}{\underline{N}} = \underset{\text{initial}}{\underline{G^0}} + \underset{\text{direct}}{\underline{G^1}} + \underset{\text{indirect}}{\underline{G^2 + G^3 + G^4 + \dots}}$$

Indirect contributions are often greater than direct ones

$$\sum_{m=2}^{\infty} G^m > G$$

$$Indirect / Direct = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(n_{ij} - i_{ij} - g_{ij})}{\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}}$$

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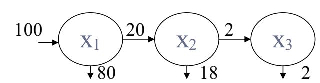
Calculating indirectness



Direct

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}$$

Integral

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ .002 & 0.1 & 1 \end{bmatrix}$$

Indirect

$$N - I - G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .002 & 0 & 0 \end{bmatrix}$$

Ind/Dir= $\frac{\sum(N-I-G)}{\sum G} = \frac{0.02}{0.3} = 0.0667$

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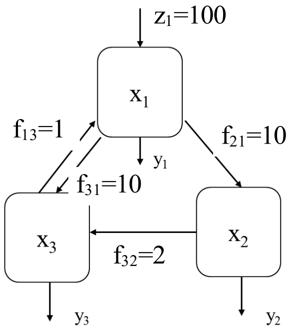
Flow analysis

Output flow environ (time forward, input driven)

Let $g_{ij}=f_{ij}/T_j$, where g_{ij} gives fraction of flow from compartment j



$$F = \begin{bmatrix} 0 & 0 & 1 \\ 10 & 0 & 0 \\ 10 & 2 & 0 \end{bmatrix}$$

T=[101 10 12]

$$G = \begin{bmatrix} 0 & 0 & .0833 \\ 0.099 & 0 & 0 \\ 0.099 & .2 & 0 \end{bmatrix}$$

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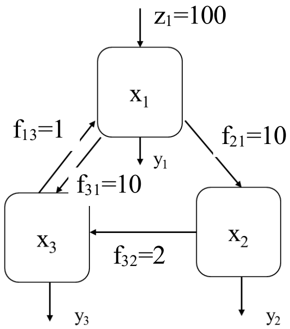
Flow analysis

Output flow environ (time forward, input driven)

Let $g_{ij}=f_{ij}/T_j$, where g_{ij} gives fraction of flow from compartment j



$$G^2 = \begin{bmatrix} 0.0083 & 0.0167 & 0 \\ 0 & 0 & 0.0083 \\ 0.0198 & 0 & 0.0083 \end{bmatrix}$$

$$G^3 = \begin{bmatrix} 0.0017 & 0 & 0.0007 \\ 0.008 & 0.0017 & 0 \\ 0.008 & 0.0017 & 0.0017 \end{bmatrix}$$

$$N = \begin{bmatrix} 1.01 & 0.0168 & 0.0842 \\ 0.1 & 1.0017 & 0.0083 \\ 0.12 & .202 & 1.01 \end{bmatrix}$$

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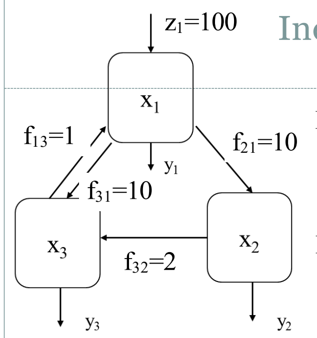
Indirectness



Direct

$$G=\begin{bmatrix} 0 & 0 & 0.0833 \\ 0.099 & 0 & 0 \\ 0.099 & 0.2 & 0 \end{bmatrix}$$

Integral

$$N=\begin{bmatrix} 1.01 & 0.0168 & 0.0842 \\ 0.1 & 1.0017 & 0.0083 \\ 0.012 & 0.202 & 1.01 \end{bmatrix}$$

Indirect

$$N-I-G=\begin{bmatrix} 0.01 & 0.0168 & 0.0009 \\ 0.001 & 0.0017 & 0.0083 \\ 0.021 & 0.002 & 0.01 \end{bmatrix}$$

$$\text{Ind/Dir}=\frac{0.077}{0.4813}=0.16$$

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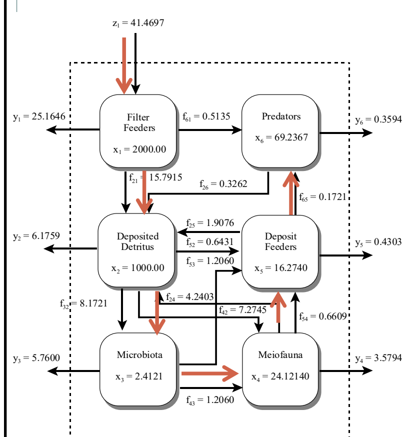
Measuring indirect flows



G =

0	0	0	0	0	0
0.3808	0	0	0.5000	0.7600	0.4758
0	0.3670	0	0	0	0
0	0.3267	0.1476	0	0	0
0	0.0289	0.1476	0.0779	0	0
0.0124	0	0	0	0.0686	0

G^2 =

0	0	0	0	0	0
0.0059	0.1853	0.1859	0.0592	0.0326	0
0.1398	0	0	0.1835	0.2789	0.1746
0.1244	0.0542	0	0.1634	0.2483	0.1554
0.0110	0.0796	0.0115	0.0144	0.0220	0.0137
0	0.0020	0.0101	0.0053	0	0

G^3 =

0	0	0	0	0	0
0.0706	0.0885	0.0136	0.0952	0.1408	0.0882
0.0022	0.0680	0.0682	0.0217	0.0120	0
0.0226	0.0605	0.0608	0.0464	0.0518	0.0258
0.0305	0.0096	0.0054	0.0415	0.0615	0.0379
0.0008	0.0055	0.0008	0.0010	0.0015	0.0009

G^4=

0	0	0	0	0	0
0.0348	0.0401	0.0348	0.0552	0.0733	0.0421
0.0259	0.0325	0.0050	0.0349	0.0517	0.0324
0.0234	0.0390	0.0145	0.0343	0.0478	0.0288
0.0041	0.0173	0.0152	0.0096	0.0099	0.0046
0.0021	0.0007	0.0004	0.0028	0.0042	0.0026

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DIRECT FLOWS

G =

0	0	0	0	0	0
0.3808	0	0	0.5000	[illegible]	0.4758
0	0.3670	0	0	0	0
0	[illegible]	[illegible]	0	0	0
0	0.0289	0.1476	0.0779	0	0
0.0124	0	0	0	0.0686	0

G^2 =

0	0	0	0	0	0
0.0059	0.1853	0.1859	0.0592	0.0326	0
0.1398	0	0	0.1835	0.2789	0.1746
0.1244	0.0542	0	0.1634	0.2483	0.1554
0.0110	0.0796	0.0115	0.0144	0.0220	0.0137
0	0.0020	0.0101	0.0053	0	0

G^3 =

0	0	0	0	0	0
0.0706	0.0885	0.0136	0.0952	0.1408	0.0882
0.0022	0.0680	0.0682	0.0217	0.0120	0
0.0226	0.0605	0.0608	0.0464	0.0518	0.0258
0.0305	0.0096	0.0054	0.0415	0.0615	0.0379
0.0008	0.0055	0.0008	0.0010	0.0015	0.0009

G^4=

0	0	0	0	0	0
0.0348	0.0401	0.0348	0.0552	0.0733	0.0421
0.0259	0.0325	0.0050	0.0349	0.0517	0.0324
0.0234	0.0390	0.0145	0.0343	0.0478	0.0288
0.0041	0.0173	0.0152	0.0096	0.0099	0.0046
0.0021	0.0007	0.0004	0.0028	0.0042	0.0026

INTEGRAL FLOWS

N =

1.0000	0	0	0	0	0
0.5369	1.3885	0.2775	[illegible]	[illegible]	0.6606
0.1971	[illegible]	[illegible]	0.2863	0.4039	0.2425
0.2045	[illegible]	0.2533	1.2971	0.4192	0.2516
0.0605	0.1565	0.1904	0.1659	1.1241	0.0745
0.0165	0.0107	0.0131	0.0114	0.0771	1.0051

TOTAL INDIRECT FLOWS

N-I-G =

0	0	0	0	0	0
0.1561	0.3885	[illegible]	[illegible]	0.3406	0.1848
0.1971	0.1426	0.1019	[illegible]	[illegible]	0.2425
0.2045	0.2021	[illegible]	[illegible]	0.4192	0.2516
0.0605	[illegible]	0.0428	0.0879	0.1241	0.0745
0.0042	0.0107	0.0131	0.0114	0.0085	0.0051

Indirect/direct=
sum(sum(N–I–G))/sum(sum(G)) =

$$\frac{5.0523}{3.2932} = \mathbf{1.5341}$$

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When Indirect/Direct> 1, this leads to the property of **Dominance of Indirectness** – one of the key results of ecological network analysis and insights into understanding the role of networks on system organization.

Indirectness increases with increasing:

- connectivity
- cycling
- system order
- direct effects

Make the direct observation, but analyze the whole system.
Direct observations give less than half the story.

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Similar treatment is possible for input oriented (diet) flows

$$N' = \sum_{m=0}^{\infty} G'^{m} \equiv (I - G')^{-1}$$

N′ is the INTEGRAL output flow matrix since it includes direct and all indirect flows

N maps output into throughflow along all pathways

$$T = yN'$$

$$T = y\left(\sum_{m=0}^{\infty} G'^{m}\right)^{-1} \equiv y(I - G')^{-1}$$

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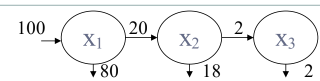

The inflow (time backward, output driven) fractions are given by g'_{ij}

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 20 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \qquad z = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$$G' = \begin{bmatrix} 0 & 0 & 0 \\ {}^{20}/_{20} & 0 & 0 \\ 0 & {}^{2}/_{2} & 0 \end{bmatrix}$$

$$g'_{ij} = \frac{f_{ij}}{T_i}$$

$$G' = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

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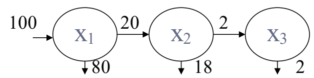
Powers of G' give flow transfers along higher order pathways.

G'^2 gives the fraction of flow entering i that arrived in two steps from j.

$$G'^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$



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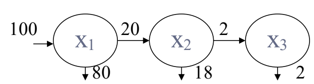
Continuing:

G'^3 gives the fraction of flow entering i that arrived in two steps from j.

$$G'^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



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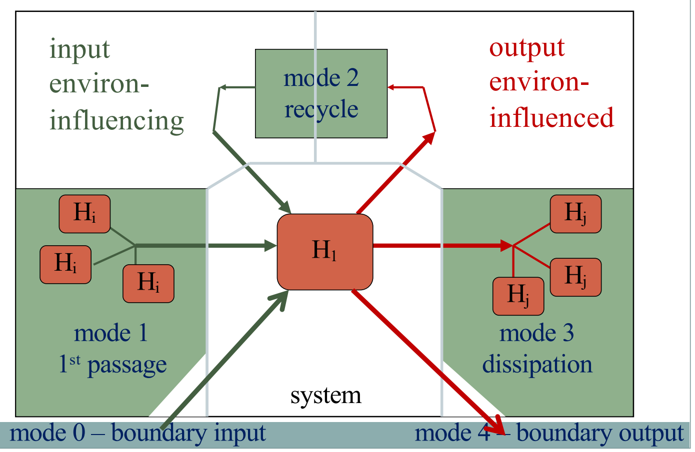
Proposition 3: Environs form a partition of the system.



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Total system throughflow

$$TST = f_{ij}^{(0)} + f_{ij}^{(1)} + f_{ij}^{(2)}$$

Total system throughflow = boundary flow + first passage flow + cycled flow

Finn Cycling Index (FCI)

$$FCI = \frac{f_{ij}^{(2)}}{TST}$$

Finn 1976, 1980

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$$FCI = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{n_{ii}-1}{n_{ii}}\right)n_{ij}z_j}{\sum_{i=1}^{n}\sum_{j=1}^{n}n_{ij}z_j}$$

N =

1.0000	0	0	0	0	0
0.5369	1.3885	0.2775	0.7800	1.1006	0.6606
0.1971	0.5096	1.1019	0.2863	0.4039	0.2425
0.2045	0.5288	0.2533	1.2971	0.4192	0.2516
0.0605	0.1565	0.1904	0.1659	1.1241	0.0745
0.0165	0.0107	0.0131	0.0114	0.0771	1.0051

$f^{(2)}$ = TSTc = 9.2082 kcal $m^{-2}d^{-1}$
TST = 83.5835 kcal $m^{-2}d^{-1}$

% cycled = TSTc/TST=9.2082/83.5835 = 0.1102 = 11.02%

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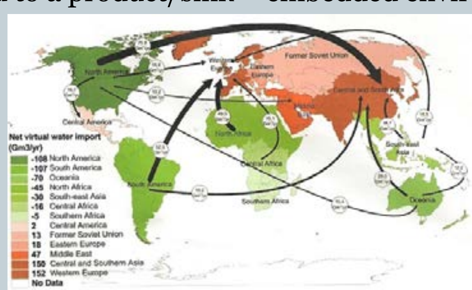
Tracing flows through networks

- Virtual or embodied flows (water, energy, minerals)
- Outward from a source – release of substance
- Inward to a product/sink – embedded environment



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THANK YOU FOR YOUR ATTENTION

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