Proposed changes in the mathematical notation used for **Ecological Network Analysis** for clearer communication

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Introduction

Ecological Network Analysis (ENA) is a family of methods (Fig. 1) to investigate

- Whole (eco)system organization
- Effects of connectivity & indirect effects
- Relative importance of nodes, groups, etc.
- Environment or environs

ENA is applied to network models of energy or matter flow through and storage in an (eco)system (Fig. 2)

40+ years of development by multiple investigators (e.g., Patten and Ulanowicz)

The lack of clear and consistent notation is a barrier for new researchers.

We propose a unified notation with a consistent row-to-column matrix orientation.

Network Model

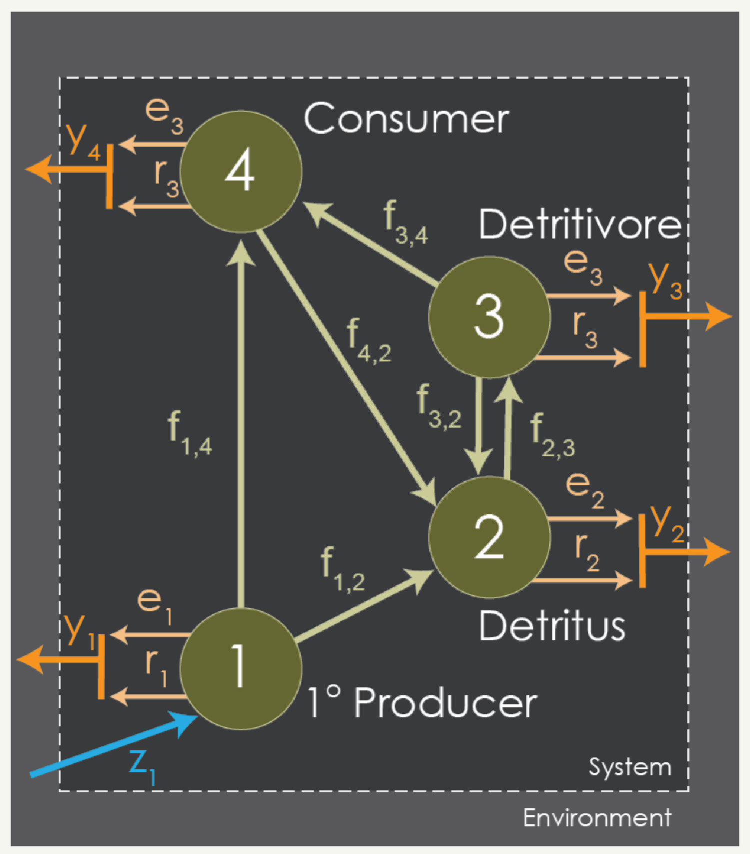

Fig. 2. Example ecosystem models. Nodes are species, groups of species, or resource pools, and edges map the flow of energy or matter.

Model Components

Symbol	Concept
$\mathbf{F}_{n\times n}$	matrix of flows from i to j (M L$^{-2 \text{ or } -3}$ T^{-1})
$z_{1\times n}$	vector of inputs (M L$^{-2 \text{ or } -3}$ T^{-1})
$r_{n\times 1}$	vector of respiration losses outputs (M L$^{-2 \text{ or } -3}$ T^{-1})
$e_{n\times 1}$	vector of export losses (M L$^{-2 \text{ or } -3}$ T^{-1})
$y_{n\times 1}$	vector of total outputs $(y+e)$ (M L$^{-2 \text{ or } -3}$ T^{-1})
$X_{n\times 1}$	vector of node storage (i.e., biomass) (M L$^{-2 \text{ or } -3}$)
$\text{Living}_{1\times n}$	logical vector indicating if a node is living
$\mathcal{S}$	stochiometric matrix whose elements indicate how the storage value of node i change when j occurs. Values can be -1, 0, or 1.
$\mathcal{M}$	$= \{\mathbf{F}, \mathbf{z}, \mathbf{r}, \mathbf{e}, \mathbf{X}, \mathbf{Living}\}$

Summary Matrices

$$\hat{\mathbf{F}} = \begin{bmatrix} \mathbf{F}_{n\times n} & e_{n\times 1} & r_{n\times 1} & 0_{n\times 1} \\ 0_{1\times n} & 0 & 0 & 0 \\ 0_{1\times n} & 0 & 0 & 0 \\ z_{1\times n} & 0 & 0 & 0 \end{bmatrix}_{(n+3)\times(n+3)}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{F}_{n\times n} & y_{n\times 1} \\ z_{1\times n} & 0_{n\times 1} \end{bmatrix}_{(n+1)\times(n+1)}$$

Proposed Notation

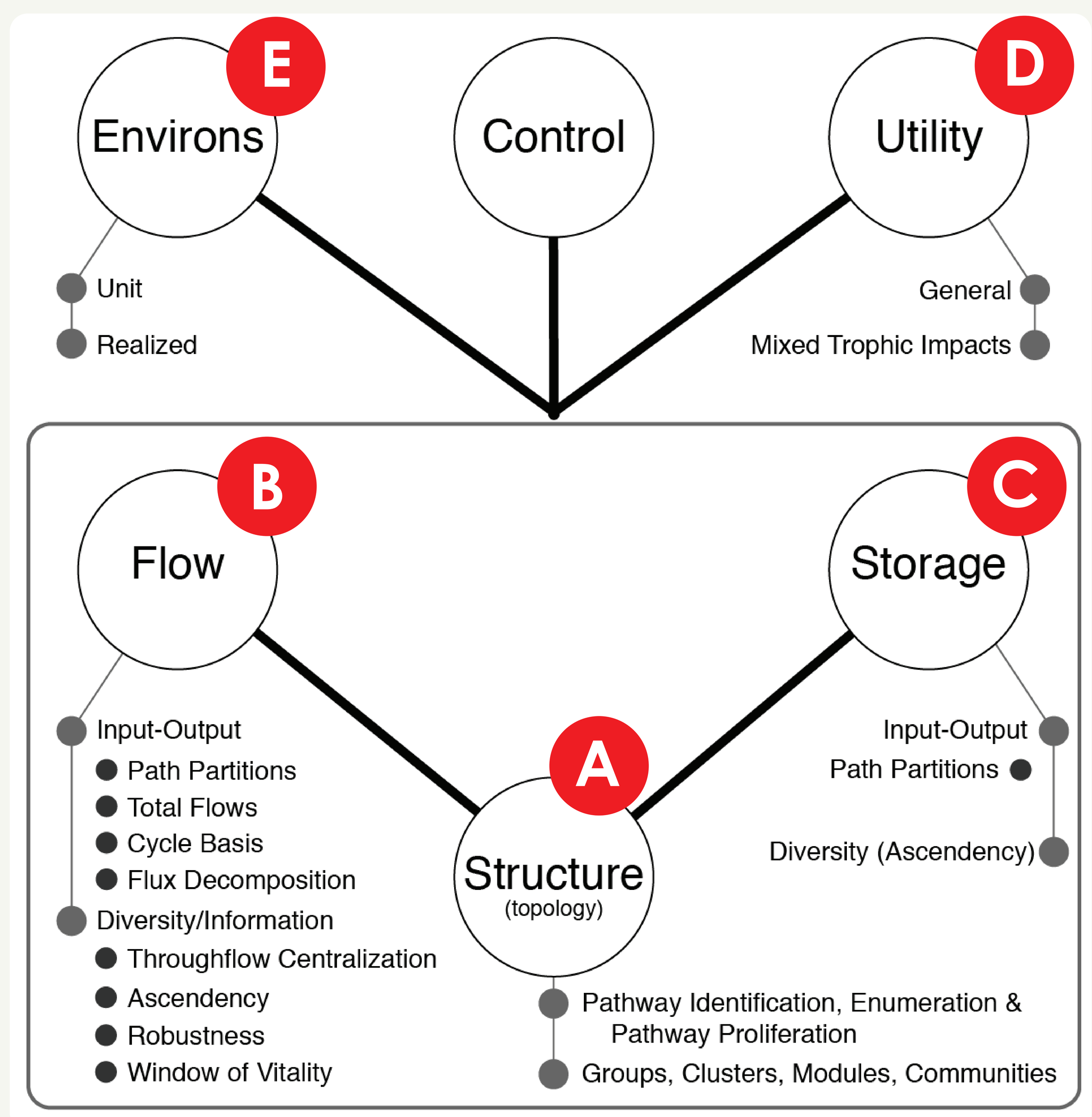

Fig. 1. Family portrait of Ecological Network Analysis methods with what Patten calls the ABCDE's of ENA. These letters correspond to the initial matrix created in each analysis

(Borrett, in prep.)

Structure Analysis (A)

Symbol	Concept	Previous
$\mathbf{A}_{n\times n}$	adjacency matrix identifying direct connections from i to j	

Flow Analysis (B)

Symbol	Concept	Previous
$T_{1\times n}$	throughflow vector (M L$^{-2 \text{ or } -3}$ T^{-1})	
$\mathbf{B}'_{n\times n}$	input-oriented direct flow intensity matrix; diet matrix; Lontief; technical coefficients (economics)	$\mathbf{G}'$
$\mathbf{B}_{n\times n}$	output-oriented direct flow intensity matrix; production/loss matrix; Augustinovics; technical coefficients	$\mathbf{G}$, $\mathbf{G}''$
$\mathbf{N}'_{n\times n}$	input-oriented integral flow intensity matrix	
$\mathbf{N}_{n\times n}$	output-oriented integral flow intensity matrix	

Storage Analysis (C)

Symbol	Concept	Previous
$\mathbf{C}'_{n\times n}$	recipient-storage normalized input-oriented direct flow intensity matrix (T^{-1}); partial turnover rates with total turnover rates on the diagonals; rate coefficients	
$\mathbf{C}_{n\times n}$	donor-storage normalized output-oriented direct flow intensity matrix (T^{-1}); partial turnover rates with total turnover rates on the diagonals; rate coefficients	
Δt	discrete time interval used to remove dimensions of $\mathbf{C}'$ and $\mathbf{C}$; selected to ensure convergence of the $\mathbf{P}'$ and $\mathbf{P}$ power series	
$\mathbf{P}'_{n\times n}$	non-dimensional storage-normalized input-oriented direct flow matrix	
$\mathbf{P}_{n\times n}$	non-dimensional storage-normalized output-oriented direct flow matrix; partial turnover rate	$\mathbf{P}''$
$\mathbf{Q}'_{n\times n}$	non-dimensional storage-normalized input-oriented integral flow matrix	
$\mathbf{Q}_{n\times n}$	non-dimensional storage-normalized output-oriented integral flow matrix	$\mathbf{Q}''$
$\mathbf{S}'_{n\times n}$	recipient-storage normalized output-oriented integral flow matrix output-oriented integral specific flow matrix (T^{-1})	
$\mathbf{S}_{n\times n}$	donor-storage normalized output-oriented integral flow matrix output-oriented integral specific flow matrix (T^{-1})	

Utility Analysis (D)

Symbol	Concept	Previous
	General	
$\mathbf{D}_{n\times n}$	direct utility matrix indicting the relationship from j to i	
$\mathbf{U}_{n\times n}$	integral utility matrix indicting the relationship from j to i	
$\mathbf{\Upsilon}_{n\times n}$	throughflow-scaled integral utility matrix (M L$^{-2 \text{ or } -3}$ T^{-1})	
	Mixed Trophic Analysis	
$\mathbf{B}'_{n\times n}$	input-oriented direct flow intensity matrix; diet matrix; Lontief; technical coefficients (economics)	$\mathbf{G}$
$\check{\mathbf{B}}_{n\times n}$	modified input-oriented direct flow intensity matrix	$\mathbf{F}$, $\mathbf{H}$
$\check{\mathbf{D}}_{n\times n}$	net impacts matrix	$\mathbf{Q}$
$\check{\mathbf{U}}_{n\times n}$	integral or mixed trophic impacts	$\mathbf{M}$

Environ Analysis (E)

Symbol	Concept
$\mathbf{E}'_k = [E'_{ij,k}]$	the k$^{\text{th}}$ unit input environ $(n\times n)$
$\mathbf{E}_k = [E_{ij,k}]$	the k$^{\text{th}}$ unit output environ $(n\times n)$
$\bar{\mathbf{E}}'_k = [\bar{E}'_{ij,k}]$	the k$^{\text{th}}$ realized input environ $(n\times n)$. This is scaled to the observed boundary flows.
$\bar{\mathbf{E}}_k = [\bar{E}_{ij,k}]$	the k$^{\text{th}}$ realized output environ $(n\times n)$. This is scaled to the observed boundary flows.

Control Analysis

Symbol	Concept
$\mathbf{CN}_{n\times n}$	re-scaled control matrix (flow)
$\mathbf{CQ}_{n\times n}$	re-scaled control matrix (storage)
$\mathbf{CR}_{n\times n}$	control ratio matrix
$\mathbf{CD}_{n\times n}$	control difference matrix
$\mathbf{CA}_{n\times n}$	control allocation matrix; the control strength i exerts on j
$\mathbf{CDep}_{n\times n}$	control difference matrix; the control strength exerted by i on j