Introduction to Single State Variable Models Lab

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Outline

I. Review Practical Programming Lab

II. Exponential Growth Models
   - Discrete time
   - Continuous time

III. Lab Tasks

Review of Practical Programming

Common Issues

Exponential Growth

Exponential Population Growth

Discrete Time – Update Rule

Use to predict future values of X

$$X(t+\Delta t) = X(t) + \Delta t(Z - Y)$$

$$X(t+\Delta t) - X(t) = \Delta t(Z - Y)$$

$$\frac{X(t+\Delta t) - X(t)}{\Delta t} = Z - Y$$

Encode with a for-loop incremented by the time step

Continuous Time – Differential Equation

$$\frac{dN}{dt} = \lim_{\Delta t \to 0} \frac{N_{t+\Delta t} - Nt}{\Delta t}$$

Rate of Change

How do we use this to predict future values?

Two Approaches

1. Analytical Solution – Exact mathematics solution

$$\frac{dN}{dt} = rN \xrightarrow{\text{integrate}} N_t = N_0 e^{rt}$$

Problem: many functions cannot be integrated

2. Numerical Approximation Algorithms

- Euler
- Revised Euler
- Runga-Kutta 2, 4
- lsoda

See reading from Shiflet and Shiflet

Numerical Approximation: Euler Algorithm

Finite Difference Method

```
growth_rate = 0.10
population(0) = 100
growth(t) = growth_rate * population(t - Δt)
population(t) = population(t - Δt) + growth(t) * Δt
```

Explicit time step

Projections

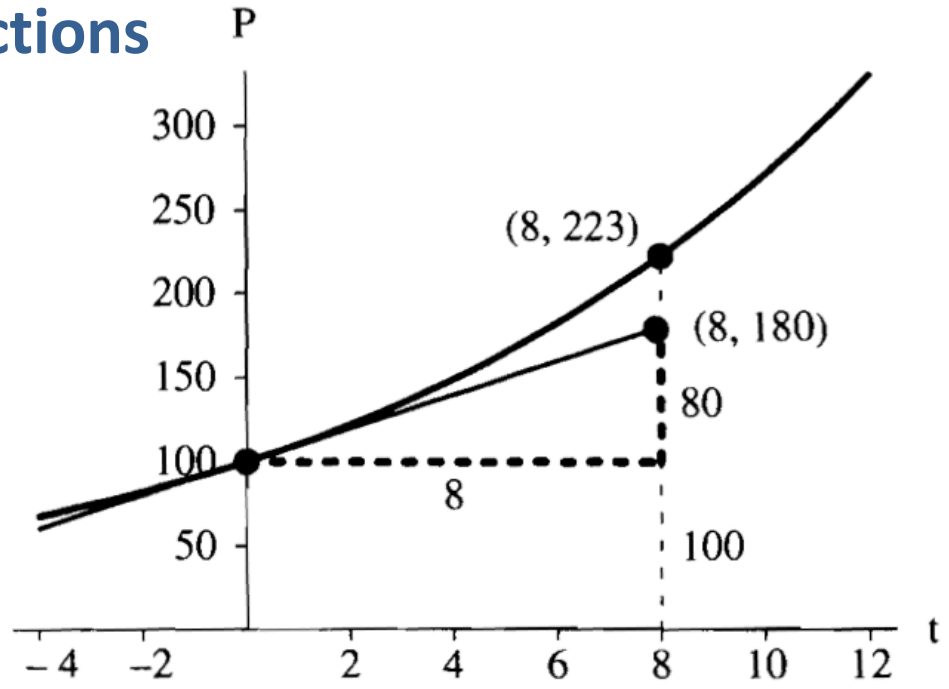


Figure 5.2.1 Actual point, (8, 223), and point obtained by Euler's Method, (8, 180)

Generates Error

Dependent on time step, rate of change, and direction of curve

Estimating Error

If we know the exact or true solution (**Na**), then we can compare it to the numerical approximation (**Np**)

Root Mean Square Error

$$RMSEP = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Np_i - Na_i)^2}$$

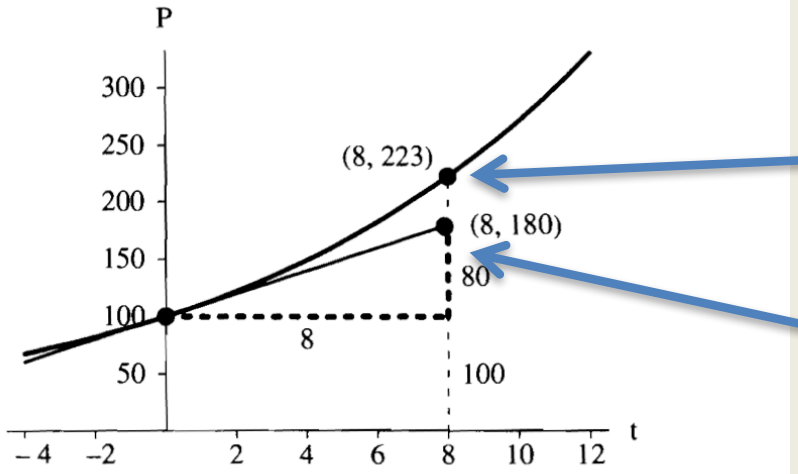


Figure 5.2.1 Actual point, (8, 223), and point obtained by Euler's Method, (8, 180)

Na(i)

Np(i)

Numerical Approximation: lsoda

- Industrial strength solver
- Uses a look forward and look backward approach
- Variable time step

Lab Activities

Please see lab manual for complete instructions

Tasks

1. Discrete Time Exponential Growth Model (EGM)
   - Code
   - Sensitivity Analysis
2. Forrester Diagram of Continuous Time EGM
3. Plot Exact solution of EGM
4. Numerical Approximation of EGM solution
   - Use deSolve package
   - Use/Modify provided R scripts
   - Estimate numerical error
   - Euler & lsoda
5. Forrester Diagram of Logistic Growth Model
6. ~Repeat above for Logistic Growth Model
7. Challenge Problem: Schaffer Equation for fishing

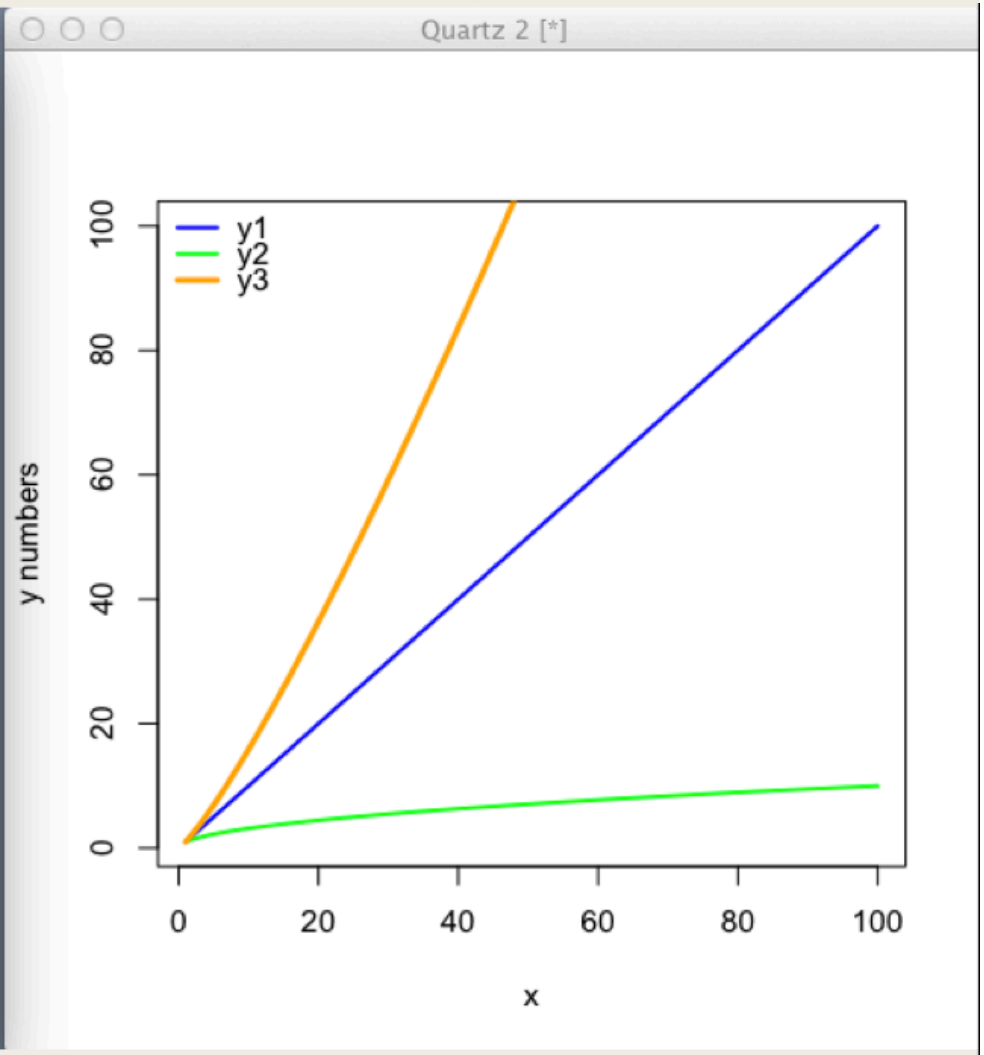
Coding Tip

Multiple Lines on a Graph

Example_Plot_multiple_lines.R

<functions> Help search

```r
x=1:100
y1 = x
y2 = x^(1/2)
y3 = x^(1.2)

# first plot - builds plot window
plot(x,y1,col="blue",type="l",lwd=2,
   ylab="y numbers")

# add another line
points(x,y2,col="green",type="l",lwd=2)

# add a thid line
points(x,y3,col="orange",type="l",lwd=3)

# create a legend
legend("topleft",
  legend=c("y1","y2","y3"),
  col=c("blue","green","orange"),
  lwd=c(2,2,3),
  bty="n")

# note that the first plot call sets the x & y axis limits.
# These may need to be adjusted.
```



deSolve Package

http://desolve.r-forge.r-project.org

Laboratory Report Format

- 1 paragraph introduction restating the laboratory objectives
- For each task
  - Briefly describe the task
  - Describe the action you take (methods)
  - Results: present evidence of task completion (usually plots)
  - Discuss the interpretation of your result and/or its ecological interpretation (when appropriate). Answer any specific questions asked in the lab manual.
- Place a copy of your R code in an appendix
  - Please practice neat coding – formatting & comments

Tasks (two weeks)

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Goal
Complete to task 4 today