Exercise 2: Propositional Logic as an Interpretive Tool

Introduction

Like the Concept Mapping homework, this exercise introduces propositional logic as a tool to extend your critical thinking toolbox. While propositional logic is a standard part of philosophy and computer science curriculum, it is a generic analytical tool. It is often useful for mapping out and interpreting the logic inherent in the complicated relationships described in ecological literature (Ford 2000).

Propositional Logic

Both classic and contemporary literature are replete with implicative relationships expressed as causes and effects, correlations, regressions, explaining variances, etc. If you can express such relationships in the form of a logical proposition, you not only may better understand the original point(s) being made, but you may also be able to extrapolate to further corollaries or logical consequences not brought to attention in the original article. Patten's (1998) "Ecology's AWFUL theorem: sustaining sustainability" provides an example of this (doi:10.1016/S0304-3800(98)00021-0). This is extension of thought is one goal of propositional logic, though it is a large subdiscipline of which we will scratch the surface (e.g., http://www.iep.utm.edu/prop-log/).

The form of a logical proposition is: IF x THEN y, or equivalently x "implies" y (written $x \Rightarrow y$). The "antecedent" x is said to be *sufficient* for the "consequent" y, and y is said to be *necessary* for x. If both sufficient and necessary conditions can be established for a proposition, then $x \Rightarrow y$ and $y \Rightarrow x$ also, meaning y "if and only if" (abbreviated iff) x. A "theorem" is a mathematical proposition, and in proving them mathematicians frequently are engaged in the establishment of necessary or sufficient conditions. Ecologists use these concepts also.

For example, the 1986 Hubbell and Foster article "Biology, chance, and history and the structure of tropical rain forest tree communities" states on p. 318:

> "... a necessary condition for becoming an abundant tree species [let this be y]... is having the ability to mature in both gap and nongap areas under the average disturbance regime [x].

As shorthand, let x = Realized Abundance (RA) and y = Eurytopic Maturation (EM) (i.e., the ability to mature in a wide range of environments); then the statement can be cast in the propositional form RA $\Rightarrow$ EM (sufficient condition on the left, necessary on the right). We'll further analyze this statement below, but first we need a little more machinery.

Logical propositions have *truth values*: they may be true (T) or false (F). We'll assume (since they said it) that the Hubbell & Foster proposition RA $\Rightarrow$ EM is true.

Logical propositions also involve a set of related propositional forms:

Proposition	Converse
$x \Rightarrow y$	$y \Rightarrow x$
Inverse	Contrapositive
$\sim x \Rightarrow \sim y$	$\sim y \Rightarrow \sim x$

Here, ~ denotes negation, and $\sim x$ (not x) and $\sim y$ (not y) are *complements* of x and y, respectively.

Now here's the power. A proposition and its contrapositve have the same truth value, and so do a converse and its inverse. For example, the Hubbell & Foster statement has the following cluster of associated propositional forms (with known truth values shown in parentheses):

Proposition	Converse
RA $\Rightarrow$ EM (T)	EM $\Rightarrow$ RA
Inverse	Contrapositive
~RA $\Rightarrow$ ~EM	~EM $\Rightarrow$ ~RA (T)

In words, the proposition states "Realized abundance of a tree species is sufficient to infer that the species succeeded at eurytopic maturation, and (the authors' original statement, paraphrased) eurytopic maturation is a necessary condition to realize abundance." These two different ways of saying the same thing are both true (T). The contrapositive form, also true (T), asserts that "Failure to mature eurytopically (i.e., under both gap and nongap conditions) is sufficient for non-abundance; the latter is a necessary condition for inferring that maturation was stenotopic." The online free dictionary (http://www.thefreedictionary.com/stenotopic) defines stenotopic in contrast to eurytropic as "able to adapt only to a narrow range of environmental conditions." Notice the seemingly subtle differences of meaning in the two proposition statements, which have been made deliberately complicated to illustrate how difficult it often can be to tease true meaning from narrative accounts. In the idea and data rich, and frequently verbose, texts of scientific papers, nuances of meaning can often be hard to interpret and evaluate. The formalism of propositional logic can cut through linguistic details and guarantee that, in the present example, for instance, any one of the four forms that might be stated (two in the proposition, two in the contrapositive) are true (T).

There is more. The converse of the Hubbell & Foster statement is "Eurytopic maturation is sufficient for realized abundance, and the latter is necessary to infer eurytopic maturation." Question: Is this statement true (T) or false (F)? Careful reflection should soon lead you to the conclusion that it is false (F). Why? This means that the inverse, which on the surface seems true, is also false (F): "Unrealized abundance is sufficient to infer stenotopic (non-eurytopic) maturation; stenotopic maturation is necessary for a tree species not to realize abundance." Subtle, right?

So, the hope is this exercise will empower you to wrestle with the kinds of logically contorted statements that appear all too frequently in ecological literature, explicitly or implicitly, statements such as hypotheses or conclusions, or results of statistical analyses, etc. If you can express these in the form of a true (T) or false (T) logical proposition, $x \Rightarrow y$, then you will be on your way to extracting full understanding, and perhaps also to exposing flaws in the original ideas. Note the if/then structure of this previous sentence. Does its analysis in the above terms yield anything nontrivial? Look at the contrapositive first to figure out its truth value.

Assignment

1. The following example relates to the Population Regulation topic.

   Proposition:

   A population whose specific or per capita growth rate, dN/Ndt, changes with density N is said to be density-dependent and therefore regulated.

   (1) Develop notations to express this proposition symbolically.
   (2) State the associated propositional forms in words.
   (3) Determine the truth value of the proposition, as well as that of its converse, inverse, and contrapositive.

2. Repeat the above exercise for an example you find or can construct from any of the readings we have covered in class.

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References
Ford, E.D., 2000. Scientific method for ecological research. Cambridge University Press, Cambridge; New York, xix, 564 pp.
Hubbell, S.P. and Foster, R.B., 1986. Biology, chance, and history and the structure of tropical rain forest tree communities. Community ecology. Harper and Row, New York, New York, USA:314-329.