Solutions

1. $\mathrm{H}_a : \mu < 35,000$

So:

$\mu_o = 35,000$
$\mu_t = 32,000$
$\sigma = 4000$
$n = 35$
$\alpha = 0.05,\ z_\alpha = 1.645$

$$\beta = 1 - P(z < \tfrac{(35000-32000)}{4000/\sqrt{35}} - 1.645) = 1 - P(z < 2.79)$$
$$= 1 - 0.9974 = 0.0026$$

If $\alpha = 0.001,\ z_\alpha = 3.090$

$$\beta = 1 - P(z < \tfrac{(35000-32000)}{4000/\sqrt{35}} - 3.090) = 1 - P(z < 1.35)$$
$$= 1 - 0.9115 = 0.0885$$

A more stringent α (lower P(type I error)) increases the type II error rate—all else being equal.

2. $\mathrm{H}_a : \mu > 250$ (You may have chosen $\mu \neq 250$ since the problem isn't specific, but its more likely that the focus would be on whether or not the TVs exceed the advertised power requirement.)

So:

$\mu_o = 250$
$\mu_t = 260$
$\sigma = 22$
$n = 20$
$\alpha = 0.025,\ z_\alpha = 1.960$

$$\beta = P(z < \tfrac{(250\text{-}260)}{22/\sqrt{20}} + 1.960) = P(z < -0.07)$$
$$= 0.4721$$

If $n = 35$

$$\beta = P(z < \tfrac{(250\text{-}260)}{22/\sqrt{35}} + 1.960) = P(z < -0.73)$$
$$= 0.2327$$

A larger sample size decreases the type II error rate—all else being equal.

3. $\text{H}_a : \mu \neq 50$

So:

$\mu_o = 50$
$\mu_t = 49$
$\sigma = 2.2$
$n = 20$
$\alpha = 0.05,\ z_{\alpha/2} = 1.96$

$$\beta = P(z < \tfrac{(50-49)}{2.2/\sqrt{20}} + 1.96) - P(z < \tfrac{(50-49)}{2.2/\sqrt{20}} - 1.96)$$
$$= P(z < 3.99) - P(z < 0.07)$$
$$= 1.0000 - 0.5279 = 0.4721$$

Power $= 1 - \beta = 0.5279$

If $\mu_t = 51$

$$\beta = P(z < \tfrac{(50-51)}{2.2/\sqrt{20}} + 1.96) - P(z < \tfrac{(50-51)}{2.2/\sqrt{20}} - 1.96)$$
$$= P(z < -0.07) - P(z < -3.99)$$
$$= 0.4721 - 0.0000 = 0.4721$$ (big surprise, right?)

Power $= 1 - \beta = 0.5279$