Type II Error Probabilities and Power for z-tests

Formulas for β = P(type II error):

$\mathrm{H}_a : \mu > \mu_o \;\Rightarrow \beta = P(z < \frac{\mu_o - \mu_t}{\sigma/\sqrt{n}} + z_\alpha)$

$\mathrm{H}_a : \mu < \mu_o \;\Rightarrow \beta = 1 - P(z < \frac{\mu_o - \mu_t}{\sigma/\sqrt{n}} - z_\alpha)$

$\mathrm{H}_a : \mu \neq \mu_o \;\Rightarrow \beta = P(z < \frac{\mu_o - \mu_t}{\sigma/\sqrt{n}} + z_{\alpha/2}) - P(z < \frac{\mu_o - \mu_t}{\sigma/\sqrt{n}} - z_{\alpha/2})$

where μ_t is the actual population mean.

Power = 1 - β in all cases.

Exercises

1. A tire manufacturer claims that its tires last 35,000 miles, on average. A consumer group wishes to test this, believing it is actually less. The group plans to assess lifetime of tires on a sample of 35 cars and test these assumptions at $\alpha = 0.05$. If the standard deviation of tire life is 4000 miles, what is the probability of a type II error if the actual mean lifetime of the tires is 32,000 miles? What if $\alpha = 0.001$?

2. A company claims that their top television set only requires 250 microamperes of current to maintain an adequate brightness level. If the standard deviation of the required current is 22 microamperes and a sample of 20 sets is to be chosen, what is the type II error rate of the test if the true mean power requirement is 260 microamperes (assuming the test is conducted at the 0.025 level)? What if 35 sets are sampled?

3. Speedometers in autos made by a certain manufacturer are regularly checked in batches for accuracy. Each vehicle is run at a speedometer reading of 50mph and speed measurements are taken via an independent source. If the company regularly samples in batches of 20, and the standard deviation of actual speeds (when the speedometer reads 50mph) is 2.2 mph, what is the power of the test, assuming $\alpha = 0.05$, when the actual mean speed is 49mph? What if the actual mean speed is 51 mph?