The Conservation of Linear Momentum

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Theoretical Discussion

The principle of conservation of momentum states that

The total momentum of an isolated system is constant.

This means that the change in the total momentum of an isolated system from any time to any other time is zero. Mathematically,

$$\Delta\vec{p} = m\Delta\vec{v} = 0$$

If there are N particles making up the system, then the vector sum of the individual momenta is conserved, so that, for an isolated system,

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 + \cdots + \Delta\vec{p}_N = 0$$

If the particles in a system collide, conservation of momentum implies that the total momentum before and after the collision is the same.

$$\vec{p}_{1,i} + \vec{p}_{2,i} + \cdots + \vec{p}_{N,i} = \vec{p}_{1,f} + \vec{p}_{2,f} + \cdots + \vec{p}_{N,f}$$

or

$$\sum_{j=1}^{N} \vec{p}_i = \sum_{j=1}^{N} \vec{p}_f$$

While both the momentum and energy of an isolated system are always conserved, it may be that some of the energy in a collision is converted to internal energy of one or more of the colliding objects. It may appear that energy is not conserved, but this is not the case. It is merely that this "lost" energy no longer has the form either of kinetic or potential energy. This internal energy is hard to keep track of and may not be directly recoverable as kinetic or potential energy because it has dissipated throughout the particles making up the system. Nonetheless, even in such collisions both the total momentum and the total energy are still conserved. Collisions in which the total momentum is obviously conserved but some part of the kinetic or potential energy is converted to internal energy of the objects making up the system are known as *inelastic* collisions. Collisions in which none of the energy is converted to internal energy, so that the conservation of energy is also obvious, are known as *elastic* collisions. Today's lab will elastic collision in one dimension.

Relationship between initial and final velocities in an elastic collision

One particular type of elastic collision is the "rear-end" collision, in which a moving object of mass m_1 and initial velocity $\vec{v}_{1i}$ rams another object of mass m_2 which is at rest. Assuming that the objects are not subject to any external forces, conservation of momentum and energy imply that

$$\begin{aligned} m_1\vec{v}_{1i} &= m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \\ \frac{1}{2}m_1 v_{1i}^2 &= \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 \end{aligned}$$

If the motion of the objects is confined to one dimension, the conservation equations can be written as

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \tag{1}$$

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \tag{2}$$

where the common factor of $\frac{1}{2}$ has been divided out of the second equation. If we solve equations 1 and 2 for v_{1f}^2, we get

$$v_{1f}^2 = v_{1i}^2 + \frac{m_2^2}{m_1^2} v_{2f}^2 - \frac{2m_2}{m_1} v_{1i} v_{2f} \tag{3}$$

$$v_{1f}^2 = v_{1i}^2 - \frac{m_2}{m_1} v_{2f}^2 \tag{4}$$

respectively. Setting equations 3 and 4 equal to one another in order to eliminate v_{1f}^2 and then solving for v_{2f}, one obtains

$$v_{2f} = \frac{2v_{1i}}{1 + \frac{m_2}{m_1}} \tag{5}$$

If one solves equation 1 for v_{1f}, one obtains

$$v_{1f} = v_{1i} - \frac{m_2}{m_1} v_{2f} \tag{6}$$

Substituting equation 5 into equation 6 and performing some algebraic re-arrangement of the factors yields

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

So that the velocities of objects 1 and 2 following the collision are given in terms of the initial velocity of object 1 as

$$v_{1f} = \left(\frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} \right) v_{1i} \tag{7}$$

$$v_{2f} = \frac{2v_{1i}}{1 + \frac{m_2}{m_1}} \tag{8}$$

We can make several quantitative observations just by inspecting these equations. We can specialize to three cases; $m_1 > m_2$, $m_1 = m_2$, and $m_1 < m_2$. Note that in all three cases, $v_{2f} > 0$, so that the struck object always moves in the same direction as the striking object's initial velocity. This is rather intuitively obvious.

1. $m_1 > m_2$

   - If $m_1 > m_2$, then $v_{1f} > 0$, which is to say that object 1 continues to move in the "forward" direction.
   - In the limit that $\frac{m_2}{m_1} \to 0$, $v_{1f} \to v_{1i}$.

- If $m_1 > m_2$, then $v_{2f} > v_{1i}$
- In the limit that $\frac{m_2}{m_1} \to 0$, $v_{2f} \to 2v_{1i}$.

2. $m_1 = m_2$

- If $m_1 = m_2$, then $v_{1f} = 0$, which is to say that object 1 stops.
- If $m_1 = m_2$, then $v_{2f} = v_{1i}$. What happens is that object 1 transfers all of its momentum and kinetic energy to object 2.

3. $m_1 < m_2$

- If $m_1 < m_2$, then $v_{1f} < 0$, which is to say that object 1 reverses direction.
- In the limit that $\frac{m_2}{m_1} \to \infty$, $v_{1f} \to -v_{1i}$.
- If $m_1 < m_2$, then $v_{2f} < v_{1i}$
- In the limit that $\frac{m_2}{m_1} \to \infty$, $v_{2f} \to 0$.

In today's experiment we will investigate all three cases.

Experimental Procedure

In this experiment, you will crash a "predator" air cart into a "victim" air cart, as shown in figure 1. You will measure the time intervals during which the "sails" of the air carts block the infrared beams of the photogates, which will enable you to calculate their speeds both before and after the collisions. To convert these times into speeds, you will need to measure the width of the sails. The speeds are then just the sail widths divided by the time intervals. You will also need to measure the masses of each of the air carts. In each of the cases under investigation, the initial speed of the victim will be zero. According to the figure, object 1 is the predator and object 2 is the victim.

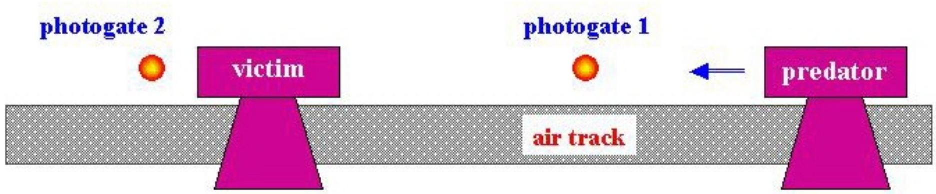

FIG. 1: Experimental setup for the measurement of momentum conservation

For each of the three cases discussed above; $m_1 > m_2$, $m_1 = m_2$, and $m_1 < m_2$; you will crash air cart 1 into air cart 2. Air cart 2 will be initially at rest, innocent and unsuspecting, at some point directly in between the two photogates, as air cart 1 bears down upon it. Air cart 1 will pass through the left-most photogate prior to the collision, will strike air cart 2, and then will either move forward or backward depending on the relative masses of air carts 1 and 2. After the collision, air carts 1 and 2 will both move through one or the other of the photogates. It doesn't matter which photogate they go through, but it is imperative that you note the order through which they pass through some one of the photogates. This is because the timer will read back the time intervals in the order in which they were measured. Since you need to be able to correlate the times with the air carts, you must note the sequence in which the air carts move through the gates.

Using the photogate timer

We will be using the photogate timer in S_1 mode. In this mode, the timer measures the time interval during which the photogate beam is blocked. If the timer is not already in S_1 mode, you can put it in this mode by repeatedly (and gently) pushing the *function* button until the LED indicator shows that you are in the proper mode. Pay attention to which time units are being displayed. You may wish to clear the data from the timer's memory buffers. You can do this by pushing the *clear* button.

After you have taken your measurements for any given experimental trial, push the *stop* button. The timer will then sequentially display the measured time intervals, in the order in which they were recorded. If you have done everything right, you should have recorded three time intervals. After you have written down your data, push *stop* again, then clear the timer and move on to the next trial.

Data Analysis

For each of the two cases; $m_1 > m_2$ and $m_1 < m_2$;

1. Use your time interval measurements and your sail-width measurements to calculate v_{1i}^{exp}, v_{1f}^{exp}, and v_{2f}^{exp}.

2. Use your experimental value v_{1i}^{exp} to calculate v_{1f}^{pred} and v_{2f}^{vict} from equations 7 and 8.

3. Numerically compare v_{1f}^{pred} to v_{1f}^{exp}, and v_{2f}^{vict} to v_{2f}^{exp} by means of fractional discrepancies.