Minimum Spanning Trees

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Reference clrs, Chapter 23, Page 561-

Problem Statement: Minimum Spanning Tree

Given an undirected, connected, weighted graph,
$G = (V, E)$,
find a spanning tree T of G of minimum weight.

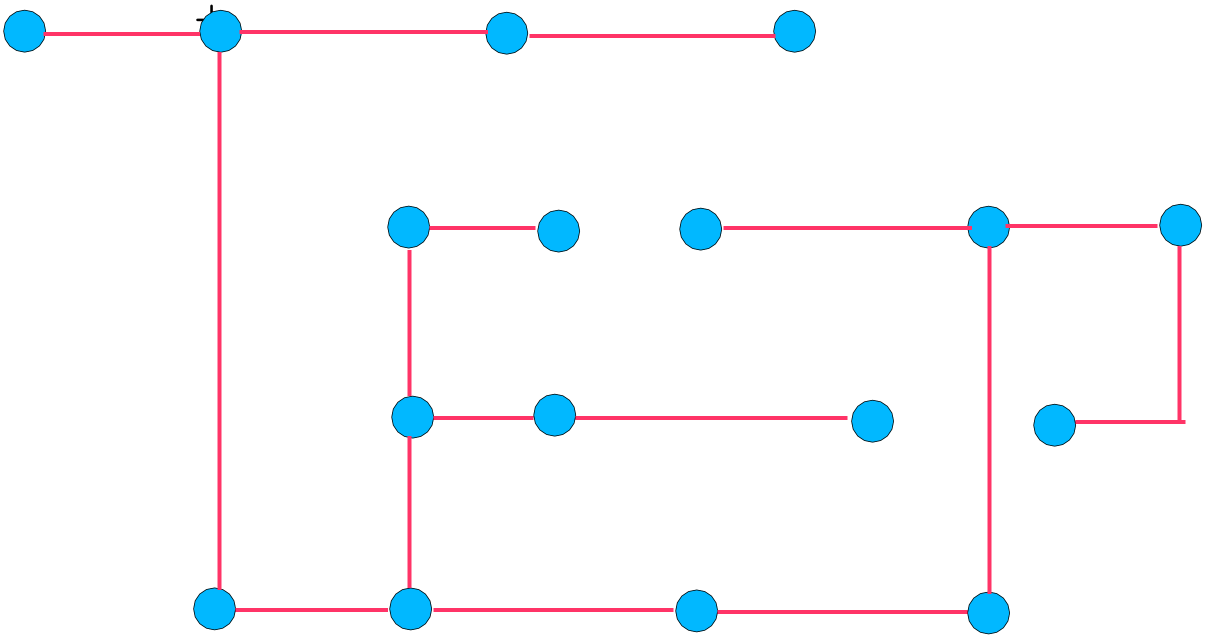
If $w(u, v)$ represents the weight of the edge (u, v) then we are interested in an acyclic subset $T \subseteq E$ that connects all the vertices and whose total weight

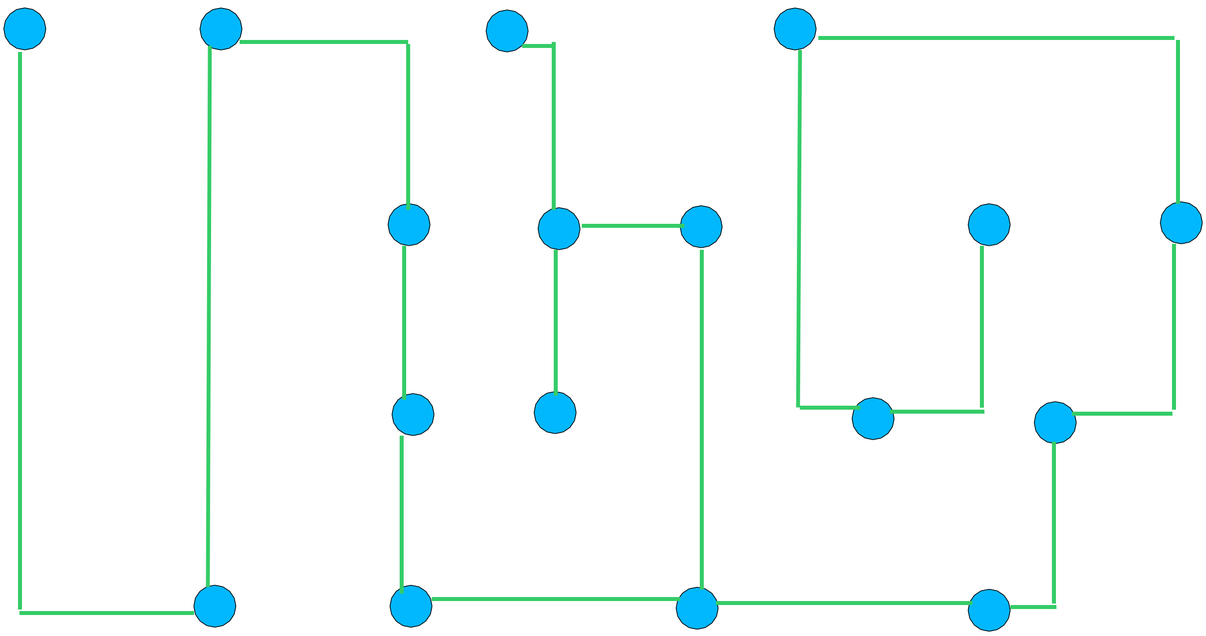
$$w(T) = \Sigma_{(u,v) \in T} w(u, v)$$

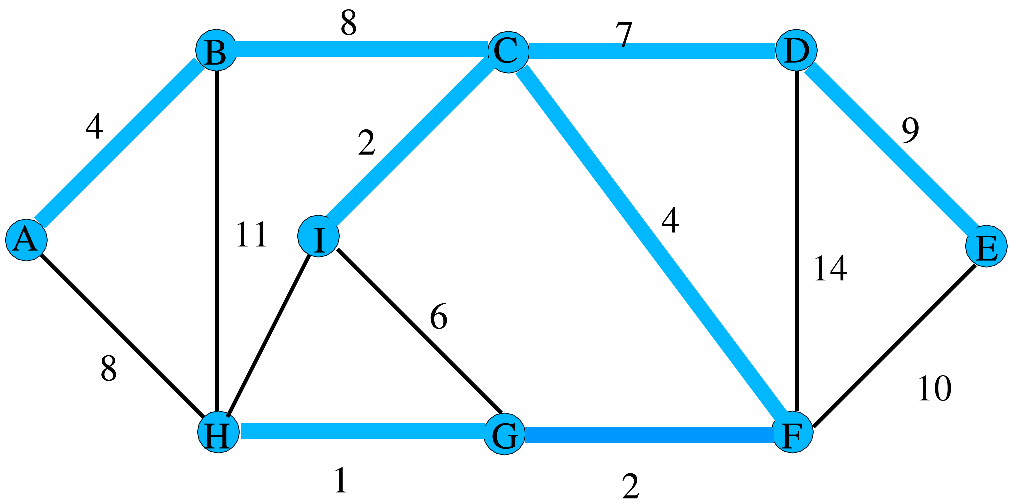
is minimized.

Example Application: Minimum Spanning Tree

In the VLSI (Very Large Scale Integrated)chip design where n pins (terminals) have to be connected to provide power (or ground) using the least amount of wire (silicon area).







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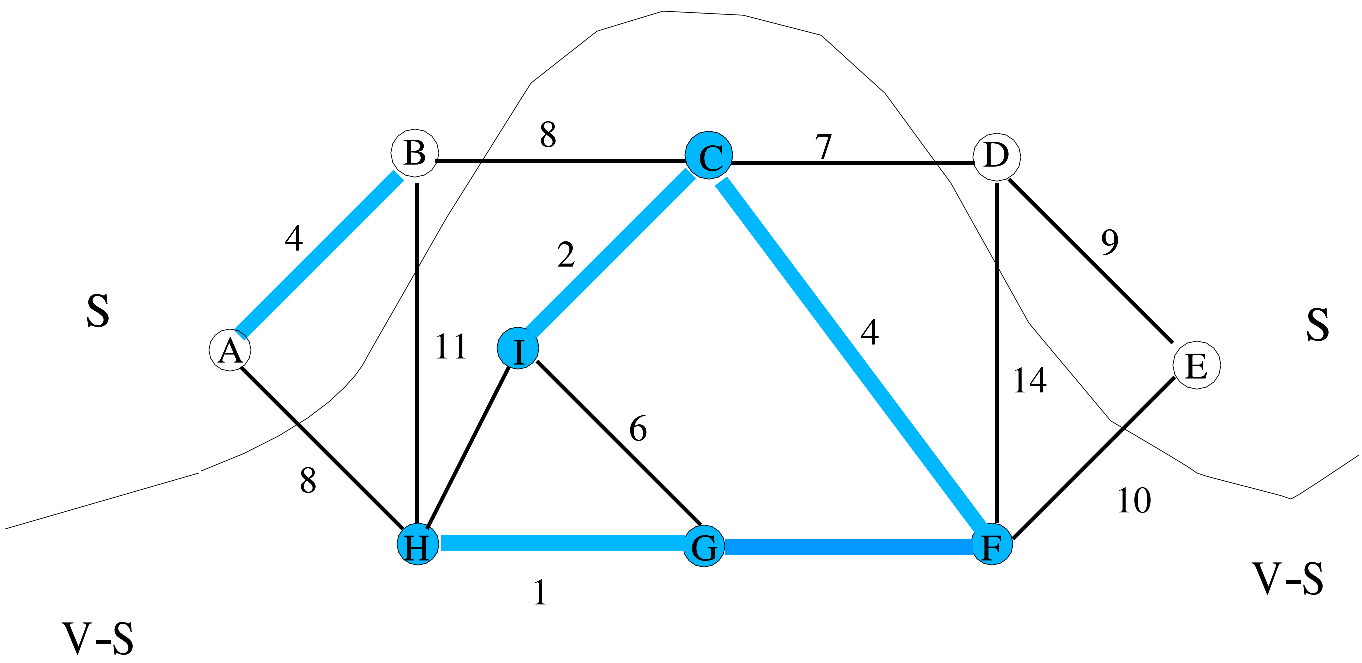
GENERIC-MST(G, w)

```
A ← φ
while A does not form a spanning tree
    do find an edge (u, v) that is safe for A
        then A ← A ∪ {(u, v)}
return A
```

Reference clrs 563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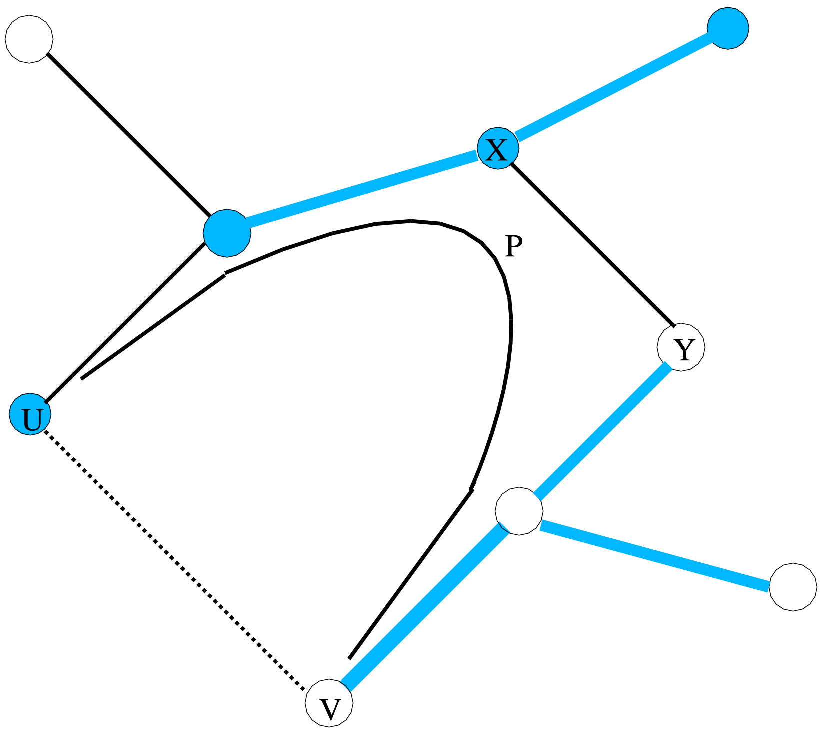
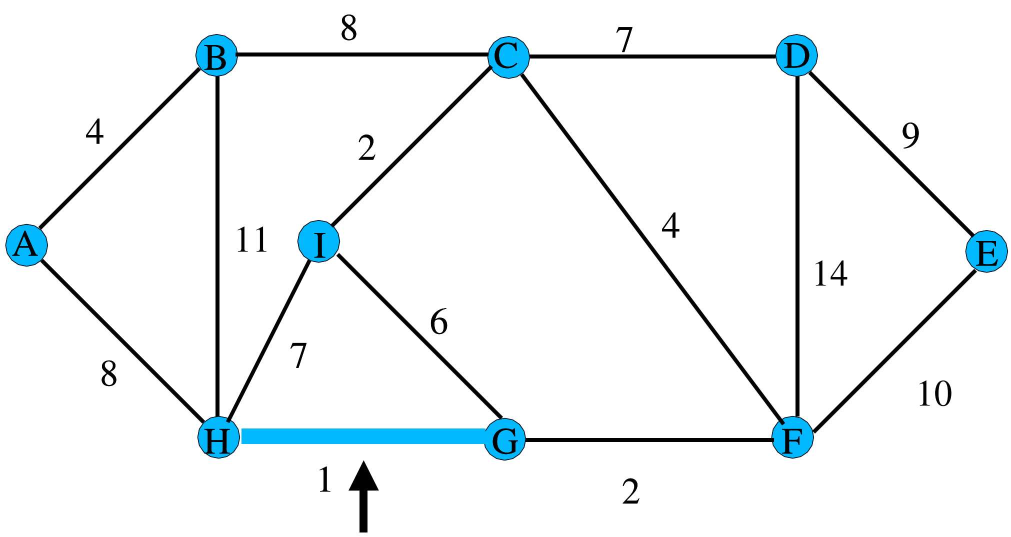


Proof of Theorem 23.1

MST-KRUSKAL(G, w)

$A \leftarrow \phi$
for each vertex $v \in V[G]$
    do MAKE-SET(v)
sort the edges of E in non-decreasing
        order by weight w
for each edge $(u, v) \in E$, taken in non-decreasing
        order by weight
    do if FIND-SET$(u) \neq$ FIND-SET(v)
        then $A \leftarrow A \cup \{(u, v)\}$
            UNION(u, v)
return A

Reference clrs 569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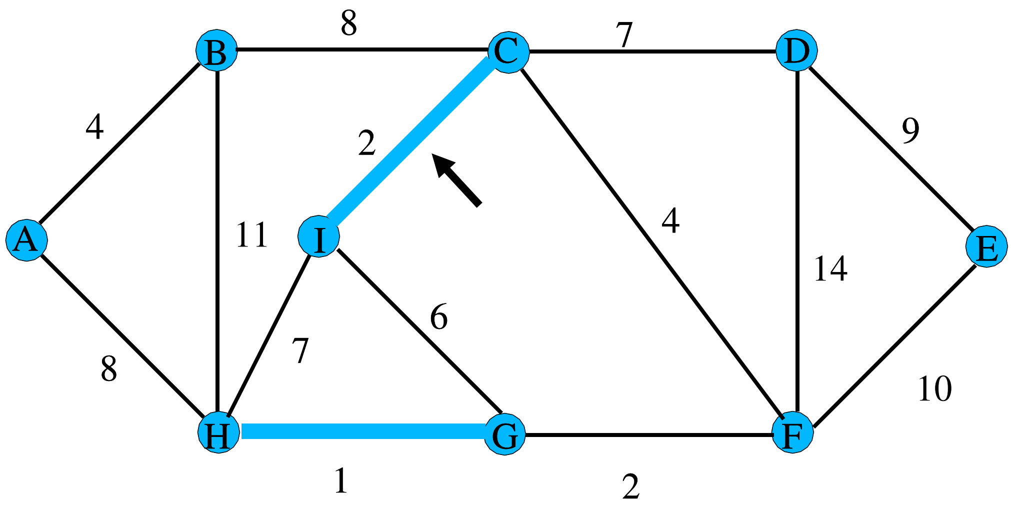
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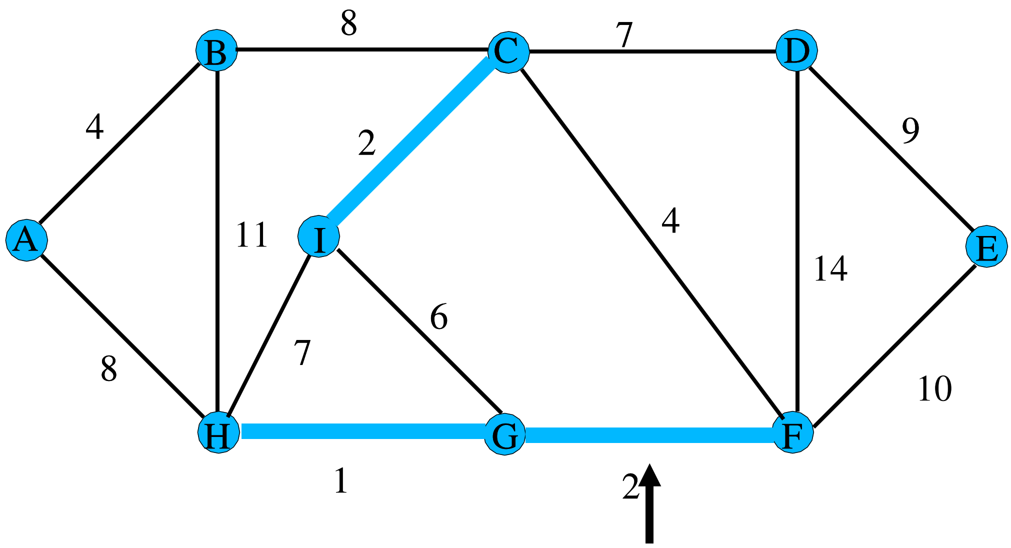

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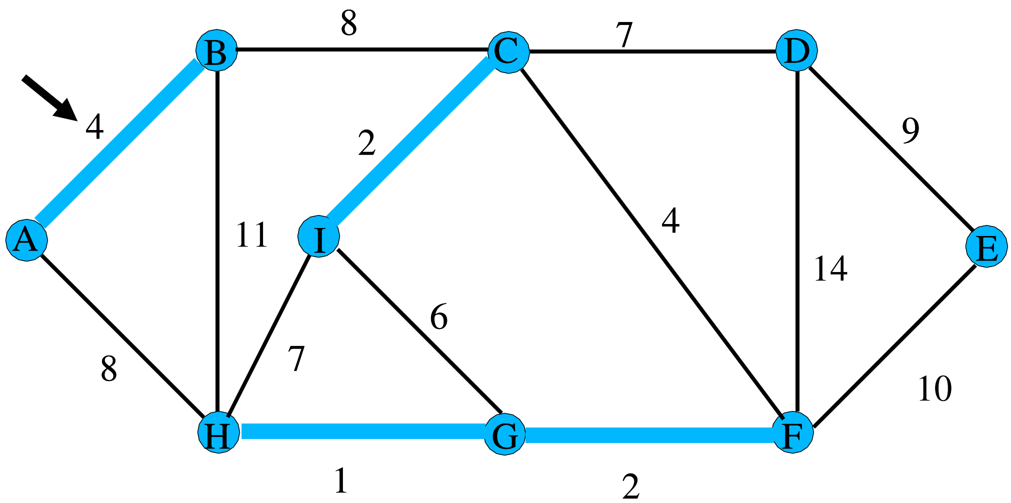
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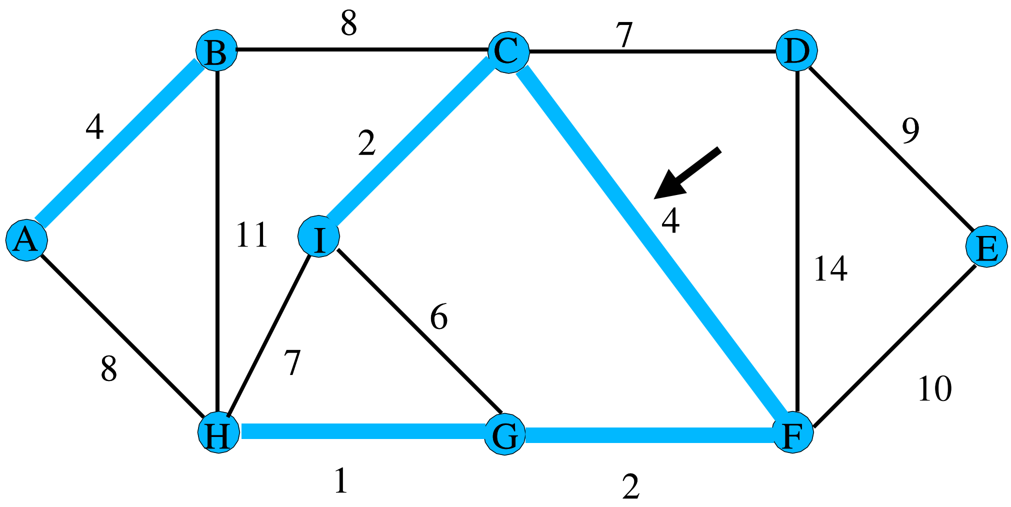
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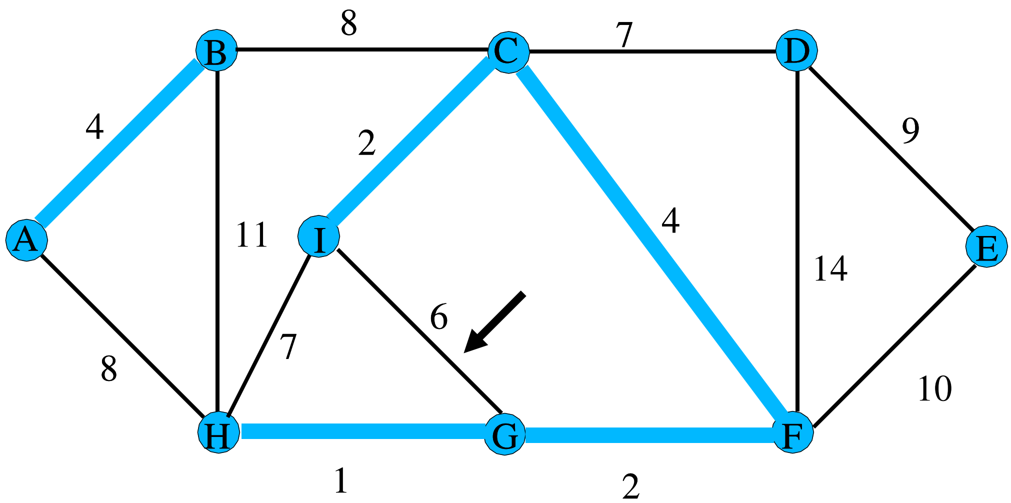

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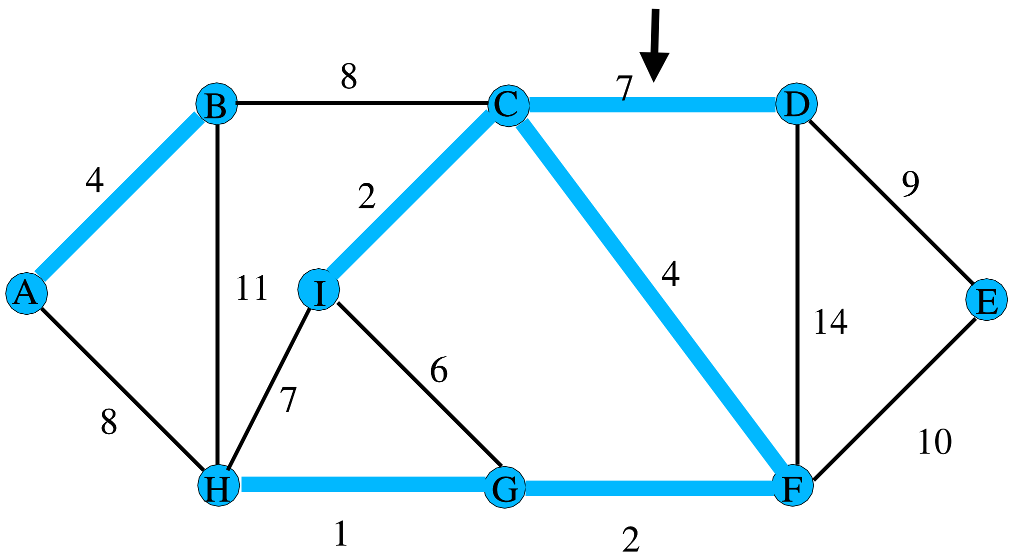

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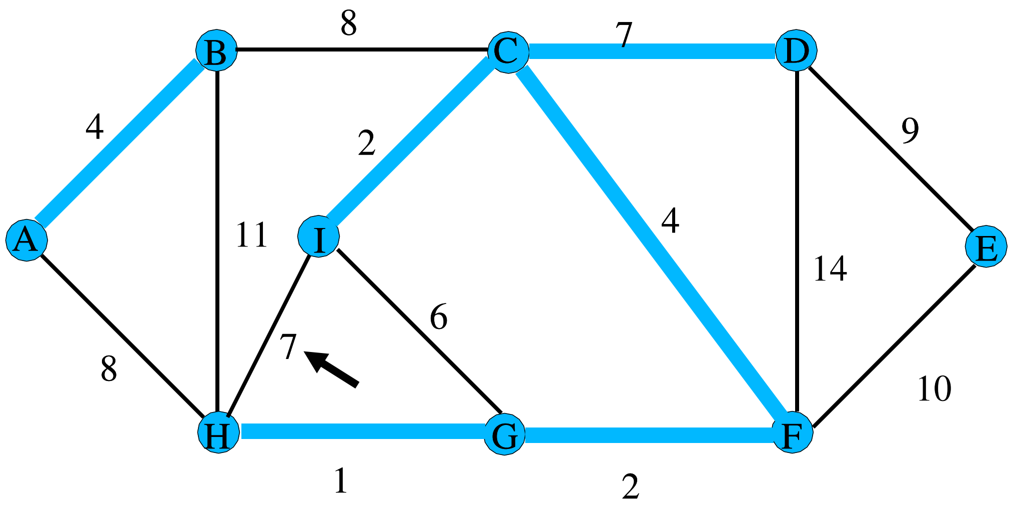

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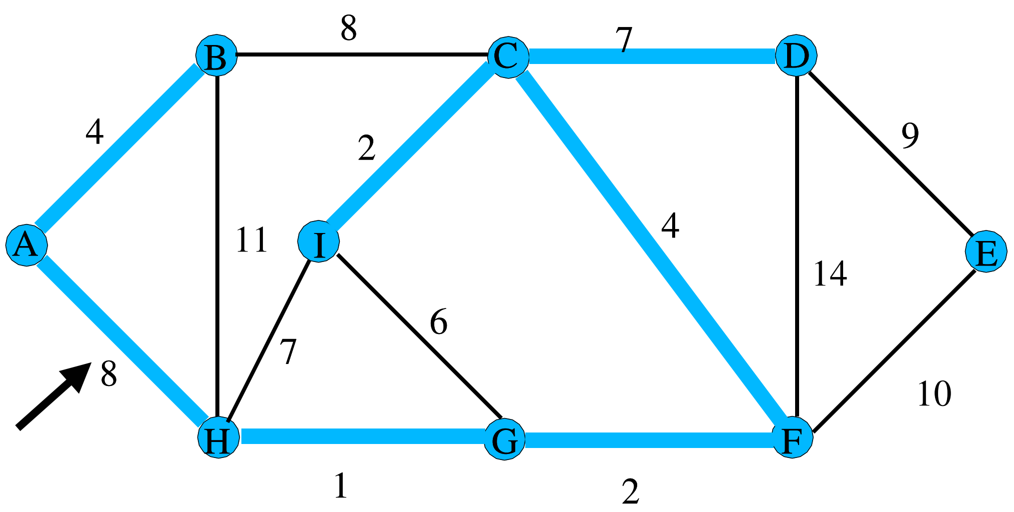

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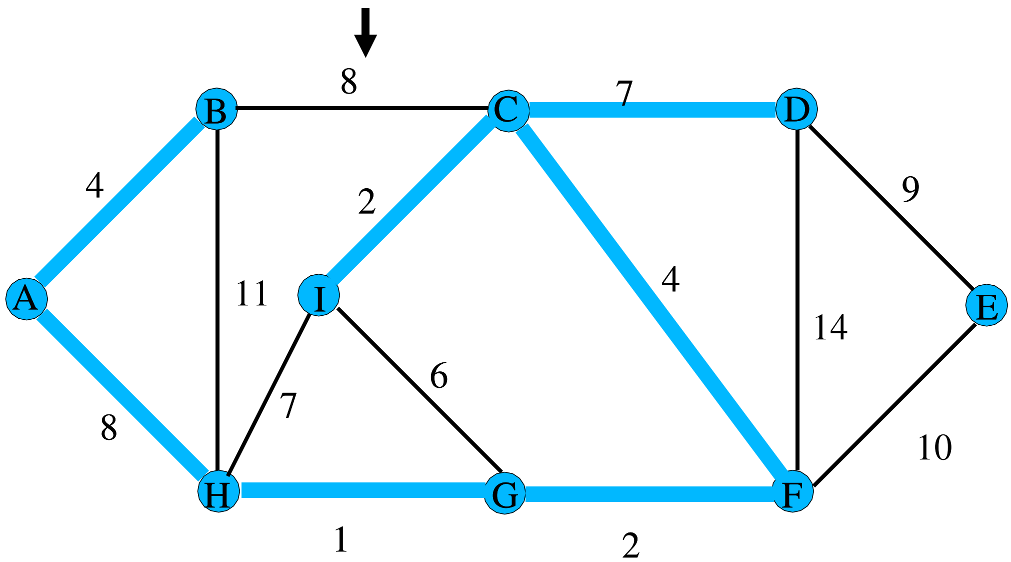
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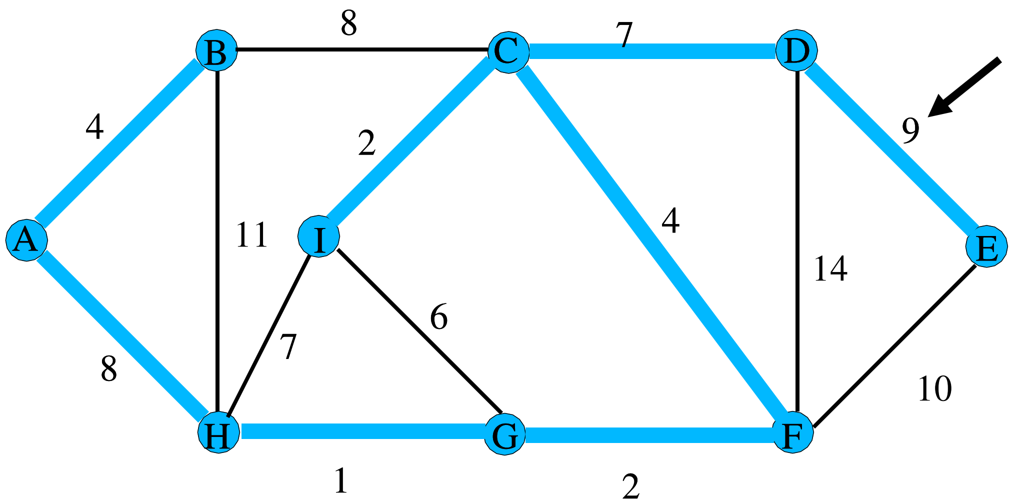
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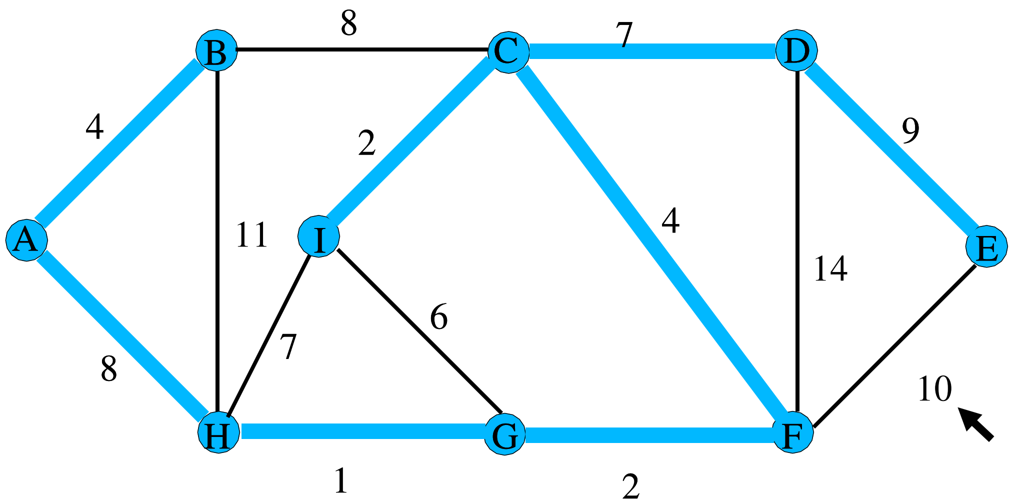
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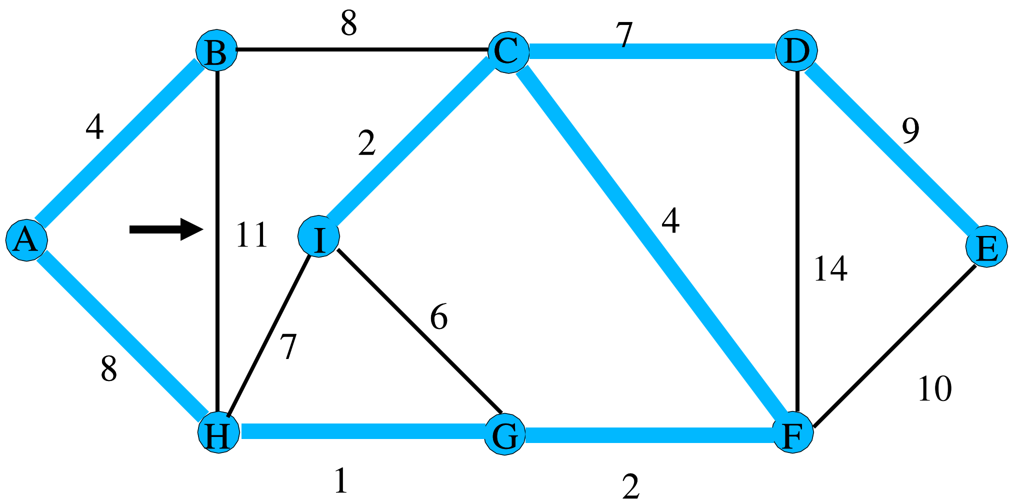
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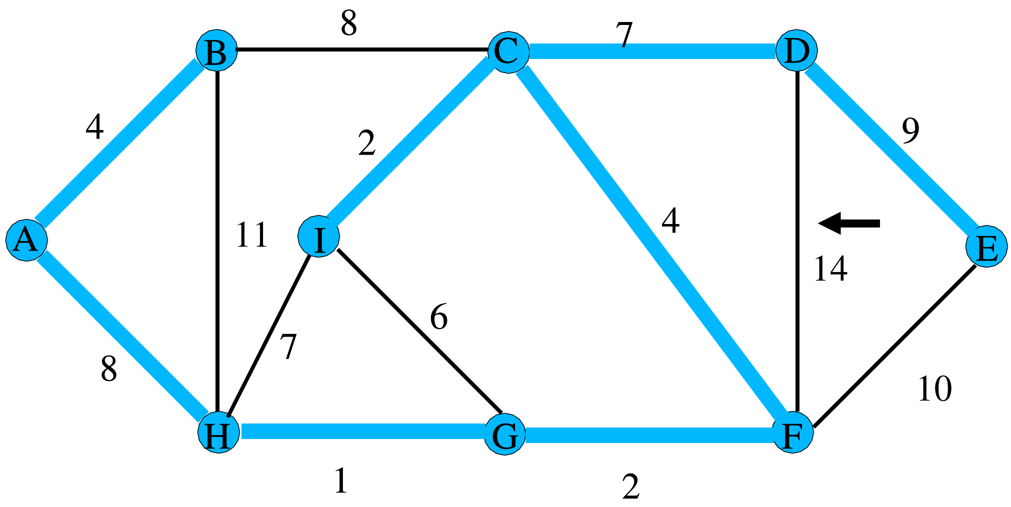

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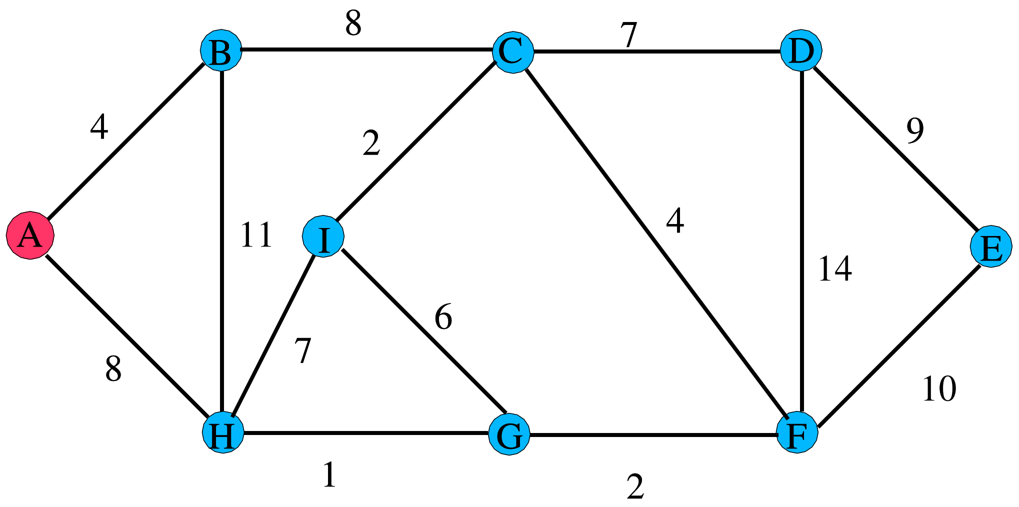
MST-PRIM(G, w, r)

```
for each u ∈ V[G]
    do key[u] ← ∞
       π[u] ← NIL
key[r] ← 0
Q ← V[G]
while Q ≠ φ
    do u ← EXTRACT − MIN(Q)
       for each v ∈ Adj[u]
          do if v ∈ Q and w(u, v) < key[v]
             then π[v] ← u
                  key[v] ← w(u, v)
```

`Reference clrs 572`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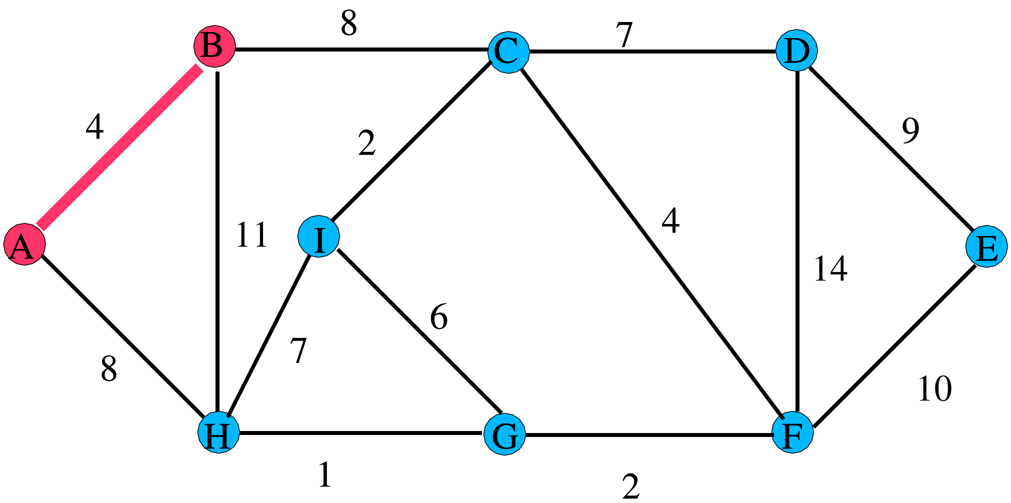

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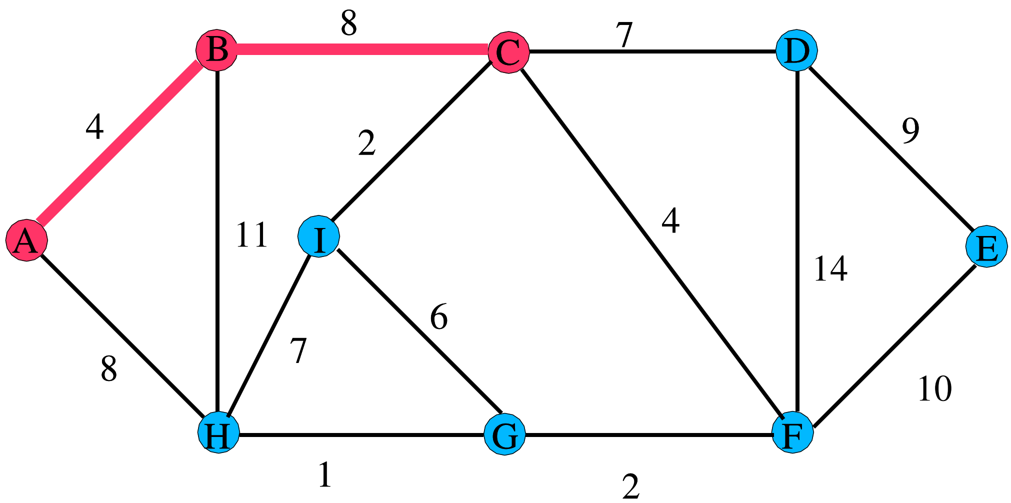

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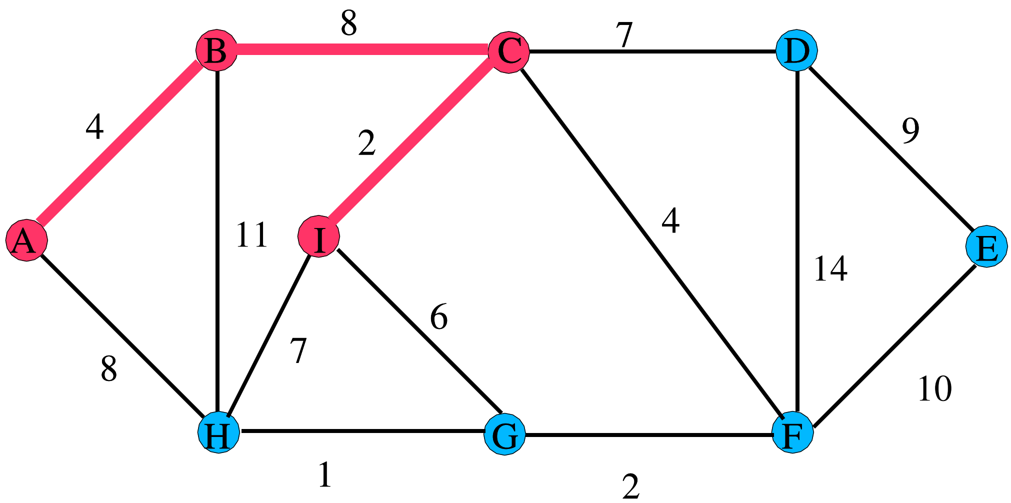

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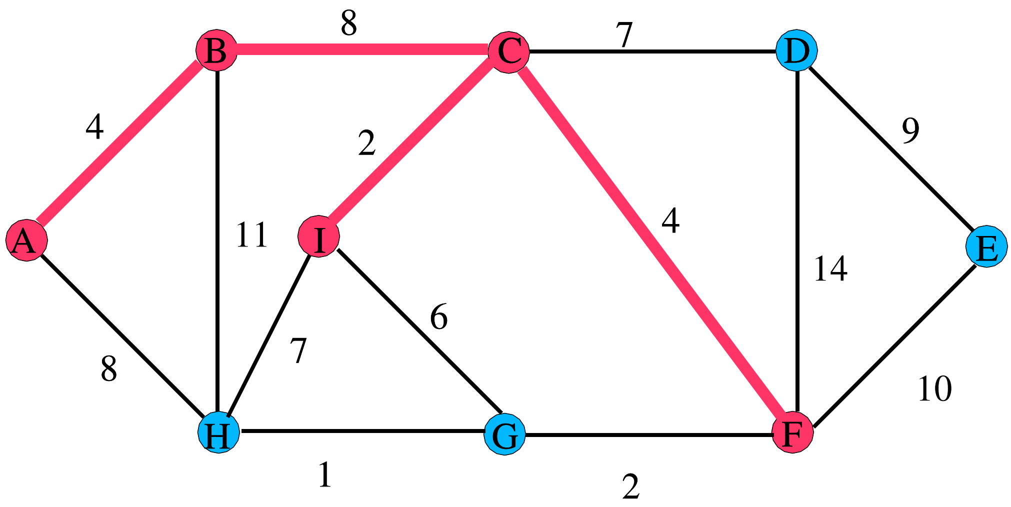
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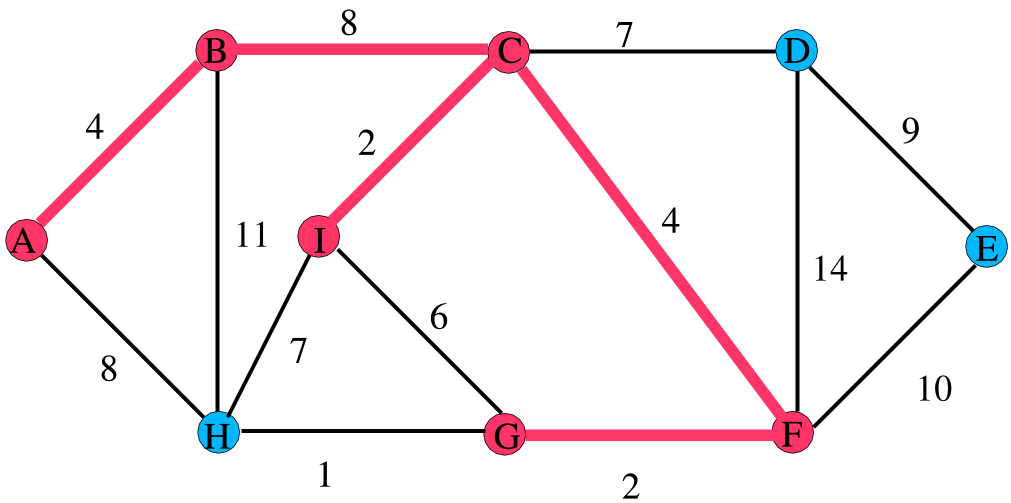
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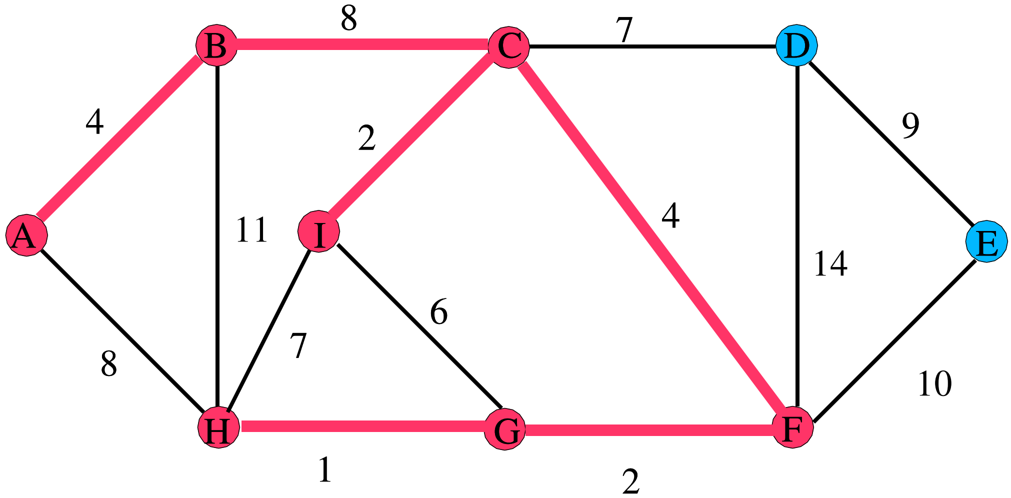
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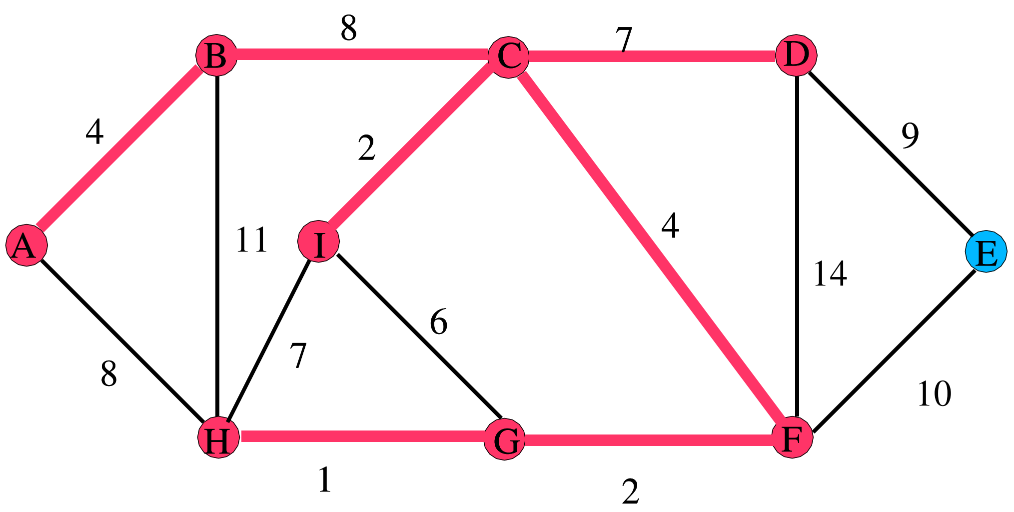
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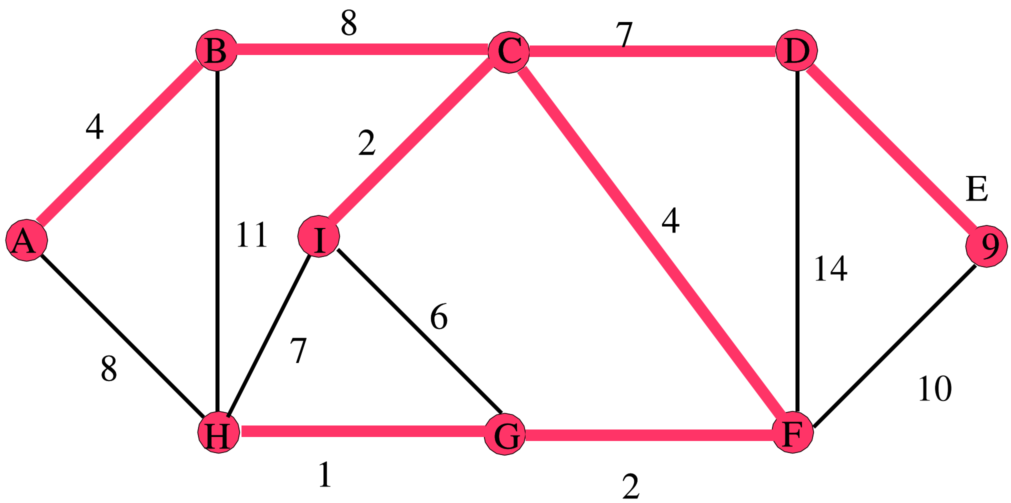
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