Math 367 Homework 3

Directions: NEATLY write all solutions on your own paper. You may discuss the problems with each other but you must write them up independently.

We define the Fourier transform of a function \( f(x) \) by

\[
\hat{f}(w) = \mathcal{F}(f(x))(w) = \int_{-\infty}^{\infty} f(x) e^{-2\pi iwx} dx
\]

and the inverse Fourier transform by

\[
f(x) = \mathcal{F}^{-1}(\hat{f}(w))(x) = \int_{-\infty}^{\infty} \hat{f}(w) e^{2\pi iwx} dw
\]

1) Show \( \mathcal{F}(e^{2\pi ikx} f(x))(w) = \hat{f}(w - k) \) and use it to find an expression for \( \mathcal{F}(\cos (2\pi kx) f(x))(w) \). Hint: Euler’s formula.

2) Show

a) \( \mathcal{F}(xf(x))(w) = \frac{i}{2\pi} \frac{d}{dw}(\hat{f}(w)) \)

b) \( \mathcal{F}(\mathcal{F}(f))(x) = f(-x) \)

3) Given that \( \mathcal{F}(e^{-ax^2})(w) = \sqrt{\frac{\pi}{a}} e^{-\frac{\pi^2 w^2}{a}} \) convince me why the convolution of two gaussians is a gaussian. Hint what is \( \mathcal{F}(f * g) \) ?

4) It is easy to show that if \( f(x) \) is a real valued function then

\[
|\int_{-\infty}^{\infty} f(x) dx| \leq \int_{-\infty}^{\infty} |f(x)| dx .
\]

The same is true for complex valued functions. Use this inequality to find a condition on \( f(x) \) that makes its Fourier transform bounded.

5) Compute \( \mathcal{F}(f(x))(w) \) for the function

\[
f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x \leq 0 \end{cases}
\]