Math 531 Homework 3

Directions: NEATLY write all solutions on your own paper. Solutions should include details like "from the definition of linearly independent we know.." You may discuss the problems with other but write up your solutions on your own.

1) Give an example of a matrix whose entries are all real numbers but whose eigenvalues are complex. Show that a positive definite matrix has positive (which implies real valued) eigenvalues.

2) Show that a symmetric matrix has real valued eigenvalues. Show that eigenvectors from distinct eigenvalues of a symmetric matrix are orthogonal.

3) Show that if $B$ a $n \times n$ symmetric matrix show $\max_{\mathbf{x} \in \mathbb{R}^n, \mathbf{x} \neq 0} \frac{\langle B(\mathbf{x}), \mathbf{x} \rangle}{\|\mathbf{x}\|^2}$ is equal to the value of the largest eigenvalue of $A$.

4) If $A$ is a real or complex $n \times n$ matrix define $\|A\| = \max_{\mathbf{x} \in \mathbb{R}^n, \mathbf{x} \neq 0} \frac{\|A(\mathbf{x})\|}{\|\mathbf{x}\|}$. Show that $\|A\|$ is equal to $\sqrt{\lambda}$ where $\lambda$ is the largest eigenvalue of $AA^*$.

5) One more to come