Show all work. Part A. Problems 1-4, 6 points each.

1. Use integration by parts to evaluate \( \int \ln(x) \, dx \)

2. What is the average value of the function \( f(x) = x^2 \) on the interval \([-2, 2]\)?

3. Does \( \int_{1}^{\infty} \frac{x}{x+1} \, dx \) converge or diverge and why? Hint: you don’t have to perform the integration to find the answer.

4. If \( f(x, y) = xe^{xy} \) find \( f_x(1, 1) \).
Part B 5-9 12 Points each

5. Use integration by parts to evaluate \( \int_{0}^{1} x^2 e^{-x} \, dx \). You may leave your answer at the evaluation bar.

6. Show the volume of a sphere of radius \( r \) is \( \frac{4}{3} \pi r^3 \) by finding the volume of the function \( y = \sqrt{r^2 - x^2} \) from \(-r\) to \( r \) rotated around the \( x \)-axis.

7. Let \( f(x, y) = 7x^2 - 2y^4 \) compute \( \frac{f(x + h, y) - f(x, y)}{h} \). What is \( \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h} \)?

8. If \( p > 1 \) find the value of \( \int_{1}^{\infty} \frac{1}{x^p} \, dx \) in terms of the variable \( p \).
9. A car dealership estimates that the total weekly sales of a car is a function of the car’s price, \( p \), and the interest rate in percent, \( i \), (here 8\% implies \( i = 8 \)) offered by the manufacturer. The approximate weekly sales are given by \( S(p, i) = 132p - 2pi - 0.01p^2 \).

a) Find an interpret \( S_i(p, i) \).

b) What would be the effect on weekly sales if the price is $9400 and the interest rate rises from 8\% to 9\%?

10. (16 pts) Show that the point \( (-1, -\frac{1}{2}) \) is a critical point for the function \( f(x, y) = x^2 - 2xy + 2y^2 + x - 5 \) and determine if it is Relative Max or Relative Min. Be sure to include all the first and second order partial derivatives needed to compute \( D \).