1. Let \( f(x) = x^4 - 12x^3 \).

   a. Use arrows to indicate the end behavior as \( x \) goes to \( \infty \) and \( -\infty \).

   b. Find all intercepts.

   \[
   \begin{align*}
   y\text{-int}: & \quad x = 0 \quad y = 0 \quad (0, 0) \\
   x\text{-int}(s): & \quad x^3(x-2) = 0 \\
   & \quad x = 0, 12 \\
   \end{align*}
   \]

   c. Use \( f'(x) \) to find the intervals where \( f(x) \) is increasing and decreasing. Then find the \( (x, y) \) coordinates of any relative max or min points.

   \[
   f'(x) = 4x^3 - 36x^2 = 0 \\
   4x^2(x-9) = 0 \\
   x = 0, 9
   \]

   Increasing on the interval(s):

   \( (0, \infty) \)

   Decreasing on the interval(s):

   \( (-\infty, 9) \)

   Local minimum point(s) (if any):

   \( (9, -2187) \)

   \[ f(9) = -2187 \]

   d. Use \( f''(x) \) to find the intervals where \( f(x) \) is concave up and concave down. Then find the \( (x, y) \) coordinates of any inflection points.

   \[
   f''(x) = 12x^2 - 72x = 0 \\
   12x(x-6) = 0 \\
   x = 0, 6
   \]

   Changes concavity at both

   Concave up on the interval(s):

   \( (-\infty, 0), (6, \infty) \)

   Concave down on the interval(s):

   \( (0, 6) \)

   Inflection point(s):

   \( (0, 0) \) and \( (6, -1296) \)

   e. On a separate page, sketch the graph of \( y = f(x) \), clearly showing all intercepts, relative max/min points, inflection points, concavity, and the general shape of the graph.
Local and absolute min

Point: (9, -2187)
2. Let \( f(x) = 4x^3 - 12x^2 + 9x \).

a. Use arrows to indicate the end behavior as \( x \) goes to \( \infty \) and \( -\infty \).

b. Find all intercepts.

\[
\begin{align*}
    \text{y-int:} & \quad x = 0 \\
    y &= 0 \\
    y &= 0 \quad (0,0) \\
    \text{x-int(s):} & \quad y = 0 \\
    \frac{4x^3 - 12x^2 + 9x}{x(4x^2 - 12x + 9)} &= 0 \\
    x &= 0, \quad 3/2, \quad 1.5 \\
    (\text{crosses} \quad x = 0) \\
    (\text{bounces} \quad f' \quad @ \quad x = 1.5) \\
\end{align*}
\]

c. Use \( f'(x) \) to find the intervals where \( f(x) \) is increasing and decreasing. Then find the \((x, y)\) coordinates of any relative max or min points.

\[
\begin{align*}
    f'(x) &= 12x^2 - 24x + 9 = 0 \\
    \text{gcf} &= 3 \\
    3(4x^2 - 8x + 3) &= 0 \\
    \text{factor:} \quad a &= 4, \quad b = -8, \quad c = -6 \\
    4x^2 - 2x - 6 &= 0 \\
    2x(2x - 1) - 3(2x - 1) &= 0 \\
    (2x - 3)(2x - 1) &= 0 \\
    x &= 3/2, \quad 1.5 \\
    (\text{local max} \quad \frac{\Delta f}{\Delta x} \quad \text{local min}) \\
    x &= \sqrt{2}, \quad x = 1.5 \\
    f' &= 0 \\
    f'' &= 0 \\
\end{align*}
\]

Increasing on the interval(s):

\((-\infty, \sqrt{2}) \cup (1.5, \infty)\)

Decreasing on the interval(s):

\((1.5, 1.5)\)

Local minimum point(s) (if any):

\((1.5, 0)\)

Local maximum point(s) (if any):

\((1.5, 2)\)

d. Use \( f''(x) \) to find the intervals where \( f(x) \) is concave up and concave down. Then find the \((x, y)\) coordinates of any inflection points.

\[
\begin{align*}
    f''(x) &= 24x - 24 = 0 \\
    24(x - 1) &= 0 \\
    x &= 1 \\
    f'' &= 0 \\
\end{align*}
\]

Concave up on the interval(s):

\((1, \infty)\)

Concave down on the interval(s):

\((-\infty, 1)\)

Inflection point(s):

\((1, 1)\)

e. On a separate page, sketch the graph of \( y = f(x) \), clearly showing all intercepts, relative max/min points, inflection points, concavity, and the general shape of the graph.
3. Let \( f(x) = e^{2x}(x + 3) \). Use product rule to find the \((x, y)\) coordinates of any relative max or min points.

\[
\begin{align*}
\frac{d}{dx} f(x) &= e^{2x} (1 + 2(x+3)) + (x+3) e^{2x} (2) = 0 \\
e^{2x} (1 + 2x + 6) &= 0 \\
e^{2x} (2x + 7) &= 0 \\
\uparrow & \quad \uparrow \\
\ne^0 & \quad x = \frac{-7}{2} = -3.5 \\
\text{Local min} & \quad (3.5, e^{-7/2})
\end{align*}
\]

4. Use Newton’s Method to approximate the solution to \( y = x^3 - 3x + 4 = 0 \), using a first guess of \( x = -2 \).
Show how you have set up the problem, and write the resulting estimates that you get until it converges in the first 6 decimal places.

\[
\begin{array}{c|c|c}
\hline
x & y = x^3 - 3x + 4 & y' = 3x^2 - 3 \\
\hline
-2 & 2 & 2 \\
\hline
\end{array}
\]

\[
x_1 = 2, \quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad \text{etc.}
\]

\[
x_2 = -2.2 \\
x_3 = -2.196215 \\
x_4 = -2.1958234
\]

Which of the following numbers would NOT make sense to use as a first guess, and why?
-3, -1, 0, 1, 2, 3

-1: \( m = 0 \), won't give you a 2nd guess
0: slope sends next guess in wrong direction
1: \( m = 0 \), won't give you a 2nd guess
2, 3, would send you near \( x = 1 \)
5. Find the dimensions of a \textit{closed} box that will \textbf{maximize volume} if the length of the box is twice the width and the surface area is 96 square inches.

\[
V = L \cdot W \cdot h
= 2w \cdot wh
= 2w^2 h
\]

\[
V(w) = 2w^2 \left( \frac{96 - 4w^2}{6w} \right)
= \frac{1}{3} (96w - 4w^3)
\]

Find the volume:

\[
V'(w) = \frac{1}{3} (96 - 12w^2)
= 32 - 4w^2 = 0
\]

\[
4w^2 = 32
w^2 = 8
w = \sqrt{8} = 2\sqrt{2} \approx 2.83
\]

\[
V' = + \quad \text{max}
\]

\[\text{Vol is maximized when}
\text{the width is about 2.83'', length is 5.66'',}
\text{and height is about 3.77''.}\]
6. Find the dimensions of a closed box with a square base that will minimize cost of materials if the volume is to be 2400 cubic inches and the top and bottom cost 5 cents per square inch and the sides cost 3 cents per square inch.

\[ V = x^2 h = 2400 \]

\[ h = \frac{2400}{x^2} \]

\[ x, h > 0 \]

\[ C = 10x^2 + 12xh \]

\[ C(x) = 10x^2 + 12x \left( \frac{2400}{x^2} \right) \]

\[ = 10x^2 + \frac{28800}{x} \]

\[ = 10x^2 + 28800x^{-1} \]

\[ C'(x) = 20x - \frac{28800}{x^2} = 0 \]

\[ 20x = \frac{28800}{x^2} \]

\[ x^3 = \frac{28800}{20} \]

\[ x = \sqrt[3]{1440} \approx 11.29 \text{ in} \]

\[ h = \frac{2400}{x^2} \approx 18.83'' \]

\[ C' = 0 \]

\[ C' \text{ neg} \quad \text{pos} \]

\[ \therefore \text{ cost is minimized when the base is approx } 11.29'' \times 11.29'' \text{ and the height is approx } 18.83''. \]
7. Use L'Hospital's Rule as appropriate to find the indicated limits. Show your work, and use notation correctly.

\[ \lim_{x \to 0} \frac{x^2}{\sin(2x) - x^2} \]

\[ \lim_{x \to 0} \frac{2x}{2 \cos(2x) - 2x} = \lim_{x \to 0} \frac{2}{4 \sin(2x) - 2} = \frac{2}{0 - 2} = -1 \]

Answer: -1

b. \[ \lim_{x \to 1} \frac{\ln(x)}{x^2 - x} \]

\[ \lim_{x \to 1} \frac{1}{x^3 - 1} = \frac{1}{4 - 1} = \frac{1}{3} \]

c. \[ \lim_{x \to \infty} \frac{\ln(x)}{\sqrt{x}} \]

\[ \lim_{x \to \infty} \frac{1}{\frac{1}{2 \sqrt{x}}} = \lim_{x \to \infty} \frac{1}{\frac{1}{2 \sqrt{x}}} = \lim_{x \to \infty} \frac{2}{\sqrt{x}} = 0 \]