Theoretical Discussion

In last week’s lab we investigated standing waves on a string. For theoretical details on the origin and nature of standing waves, please see last week’s lab write-up. In today’s lab, we will investigate standing sound waves in a column of air. There are two essential differences between the waves we will investigate today and the waves studied in last week’s lab. The first difference is that sound waves are longitudinal waves, whereas the vibrations of a string are transverse waves. A transverse wave is one whose oscillations are transverse, or perpendicular, to the direction of motion. The oscillations of a longitudinal wave, on the other hand, are parallel to the direction of the wave’s travel. A sound wave consists of a series of regions of alternating high and low pressure. The pressure along the length of the wave is, like the transverse wave, a periodic function that can be described by a superposition (a sum) of sine and cosine waves.

Moreover, just like the transverse wave, a traveling longitudinal wave can form a standing wave pattern if the appropriate boundary conditions are met. When these conditions obtain, waves reflected from one boundary interfere constructively with waves emitted from the other, so that their amplitudes add. This condition is called resonance. Shock waves, for example, are longitudinal waves. Shock waves from an earthquake can be especially destructive if they satisfy a resonance condition with some geological structure. When sound waves in a column of air achieve resonance, the amplitude (volume) of the sound is enhanced. The pipes on an organ, for example, are tuned so as to form a resonant condition with particular pitches—only these particular pitches are reinforced and hence, audible.

Today’s experiment will investigate sound waves in a column of air which is partially filled with water. The frequency of the sound wave, which will be generated by a tuning fork, is fixed. Since the product of the frequency and the wavelength equals the velocity of sound in air, which for a particular temperature and pressure is a constant; i.e.,

\[ f \lambda = v_{\text{sound}} \]  

Producing the resonance condition, which results in hearing an audible tone from the column, requires one to adjust the level of the water so as to produce a column length \( L \) that satisfies the resonance condition. Figure 1 shows one particular mode, that for which \( L = \frac{5}{4} \lambda \). You will note from this figure that the other principle difference between today’s experiment and last week’s is that the boundary conditions for the waves are now different. In last week’s lab, the resonance condition required that both ends of the string were nodes—points of zero vibration. This led to the condition that the wavelength of the allowed modes must be an integer number of half-wavelengths. In today’s lab, the boundary which is formed by the surface of the water is a node, whereas the open end of the column constitutes an anti-node—a point of maximum vibration. The fundamental consequence of these particular boundary conditions is that the allowed wavelengths are now given by

\[ L = \frac{(2n + 1)}{4} \lambda \]  

where \( L \) is the length of the air column, from the open end to the surface of the water, and \( n = 0, 1, 2, \ldots \) is an integer. In other words, resonance will occur when the column length is an odd integer number of quarter wavelengths.

Procedure

1. Measure the room’s air temperature.
2. Adjust the water level so that it is near the top of the column. You can raise and lower the water level by raising and lowering the height of the reservoir attached to the side of the column.

3. Tap the tuning fork with the mallet while holding it at the open end of the air column. Quickly lower the water level and listen for the resonances as the water level drops. Make a mental note of the approximate water levels of the different allowed modes. Record the frequency $f_1$ of the tuning fork, which is engraved on it in units of Hz.

4. Find the first $(n = 0)$ and second $(n = 1)$ resonance levels. Record the values of $L_1$ and $L_2$ for this first frequency $f_1$. Calculate the wavelength $\lambda_1$ associated with frequency $f_1$, using equation 2.

5. Repeat the previous procedures using a second tuning fork with a different frequency $f_2$.

6. Use equation 1 to calculate the speed of sound in air for frequencies $f_1$ and $f_2$. Take the average of these two values, so that your experimental speed of sound is equal to

$$ v_{\text{exp}} = \frac{v_1 + v_2}{2} \quad (3) $$

7. Using your measurement of the room's air temperature, calculate the theoretical value of the speed of sound in air according to the formula

$$ v_{\text{theo}} = 331.4 + 0.6T \quad (4) $$

where $v$ is in m/s and $T$ is in Celsius degrees.

8. Compare the theoretical result $v_{\text{theo}}$ from equation 4 to the experimental result $v_{\text{exp}}$ from equation 3 by calculating the fractional discrepancy between them.