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Show all your work. Circle the final answers.

1. (15 pts) Let $y' = \frac{y \cos x + 2xe^y}{1 - \sin x - x^2e^y}$. $y(0) = 1$.

(a) Circle all the following names the equation belongs to.

linear, non-linear, separable equation, exact equation, autonomous equation, homogeneous equation, logistic equation

(b) Solve the initial value problem.

2. (15 pts) Let $ty' + 2y = 4t^2$.

(a) Circle all the following names the equation belongs to.

linear, non-linear, separable equation, exact equation, autonomous equation, homogeneous equation, logistic equation

(b) Solve the equation.

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3. (15 pts) Let $a_3(x)y''' + \sin e^x y'' + \cos xy' + 2y = g(x)$ be a linear ODE on the open interval $(0, 1)$.
- (a) Give your conditions on $a_3(x)$ and $g(x)$ such that the ODE has a solution on $(0, 1)$.
 - (b) Further, define initial conditions at $x = 1/2$ for the ODE such that it has a unique solution on $(0, 1)$.

4. (15 pts) Find a general solution of $(x - 2)^2 y'' - 5(x - 2)y' + 8y = 0$.

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5. (20 pts) At time $t = 0$, a tank contains 5 lb of salt dissolved in 100 gal of water. Assume that water containing $1/4$ lb of salt/gal enters the tank at a rate of r gal/min and that the well stirred mixture is draining from the tank at the same rate r . $Q' = r/4 - rQ/100$ and $Q(0) = 5$.
- (a) Set up a model, with the initial condition, to describe the amount of salt in the tank at t .
 - (b) Find the amount of salt in the tank after 10 minutes.
 - (c) What is the limit of the amount of salt in the tank as $t \rightarrow \infty$?

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6. (20 pts) Find a general solution of $6y'' - 5y' + y = t + 3e^t$.

7. (10 pts) (a) Define the Wronskian of functions $f_1(t), f_2(t), \dots, f_n(t)$.

(b) Define the linear independence of functions $f_1(t), f_2(t), \dots, f_n(t)$ by using its Wronshian.

(c) Are functions $1, t, e^t$ liearnly independent? Justify your answer!

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8. (15 pts) Find a general solution of $y'' + y = \frac{1}{\cos x}$.

9. (15 pts) Solve the IVP $y'' + e^x y' + y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find the first four coefficients of a power series solution to the IVP.

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10. (20 pts) Find all series solution to $y'' + xy' + 2y = 0$ with $y(0) = 0$ and $y'(0) = 1$.