MAT 162– Dr. Z. Ye FINAL Summer 2019

Name :________________________________ ZID :________________________________

Show all work! Don't simplify your results!

1. (15 pts) Fill in the following blank spaces:

   (a) We extend the concept of a definite integral $\int_a^b f(x)dx$ to that of an improper integral $\int_c^d f(x)dx$ when
   either: the integral limits c, d are ________________________________.

   or: the integrand $f(x)$ is ________________________________.

   (b) An improper integral is convergent if the improper integral ____________________.

   (c) An improper integral does not exist, then the improper integral is ____________________.

   (d) If a point P has Cartesian coordinates (x, y) and polar coordinates (r, θ), then we have $x =$ ____________________θ and $y = r$____________________.

   (e) A sequence is an ordered/array of ____________________.

   (f) An infinite series is the ____________________ of real numbers.

   (g) If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is ____________________.

   (h) The sequence of the partial sums of the series $\sum_{n=1}^{\infty} a_n$ is $\{S_k\}_{k=1}^{\infty}$, where $S_k = \sum_{\_\_}^{\_\_} a_n$.

   (i) A series $\sum_{n=1}^{\infty} a_n$ is called absolutely convergent if ____________________ is convergent.

   (j) A series $\sum_{n=1}^{\infty} a_n$ is called conditionally convergent if ____________________ is convergent, but $f(x) = \sum_{n=1}^{\infty} |a_n|$ is ____________________.

   (k) The Maclaurin series of a function e^x is $e^x = \sum_{n=0}^{\infty}$ ______________x^n, for $-\infty < x < \infty$.

   (l) The Maclaurin series of a function $\frac{1}{1-x}$ is $\frac{1}{1-x} = \sum_{n=0}^{\infty}$ ______________x^n, for $-1 < x < 1$.

   (m) The Taylor series of a function f at a is $\sum_{n=0}^{\infty}$ ______________$(x-a)^n$.

Name :________________________________

2. (30 pts) Evaluate the following integrals:

(a) $\displaystyle\int_1^e (3+x)\ln x \, dx$

(b) $\displaystyle\int \sin t \cos^{2019} t \, dt$

(c) $\displaystyle\int \frac{dx}{(x+1)(3x-2)}$

(d) $\displaystyle\int \frac{x^2 \, dx}{\sqrt{1-x^2}}$

(e) $\displaystyle\int_1^\infty \frac{dx}{(3x+1)^{3/2}}$

Name :__________________________________

3. (10 pts) Let L be the length of the arc of the parabola $y^2 = x + 1$ from $(-1, 0)$ to $(1, \sqrt{2})$. Fill in the following blank spaces:

$$L = \int_{\_\_\_}^{\_\_\_} \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \, dy.$$

4. (10 pts) Let S be the area of the surface generated by rotating the curve $y = x^2$ $0 \le x \le 1$, about $x-$ axis. Fill in the following blank spaces:

$$S = \int_{\_\_\_}^{\_\_\_} \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \, dx.$$

5. (10 pts) Find the area of the surface generated by rotating the curve $4y = x^2$ from $(0, 0)$ to $(1, 1/4)$, about $y-$ axis.

6. (15 pts) Let $f(x) = \begin{cases} 1/3 & \text{if } 1 \le x \le 4, \\ 0 & \text{Otherwise} \end{cases}$ be the density function of a random variable X.

   (a) Sketch of the function $f(x)$.

   (b) Verify that f is a probability density function.

   (c) Find $P(0 \le X \le 2019)$.

   (b) Find the mean of the probability density function f.

Name :________________________________

7. (20 pts) Solve the following differential equations or initial value problems

(a) $\dfrac{du}{dt} = u + t.$

(b) $xy' + y - x\ln x = 0$ satisfying $y(1) = 0$.

8. (10 pts) Let $x = \sin t$ and $y = t + e^t$.

(a) Find $\dfrac{dy}{dx} =$

(b) Find $\dfrac{d^2y}{dx^2} =$

(c) Find an equation of the tangent line to the curve at the point $(0, 1)$.

Name :________________________________

9. (10 pts) (a) Let $x = t + 2$ and $y = t^2 - 3\sin t$ be a curve. Eliminate the parameter t to find a Cartesian expression of the curve.

   (b) Convert the point $(2, \pi/3)$ from polar to Cartesian coordinates.

10. (15 pts) True/False Questions.

    (a) If the series $\sum_{n=1}^{\infty} a_n = 5$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent. **T** or **F**

    (b) If the series $\sum_{n=1}^{\infty} a_n = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent. **T** or **F**

    (c) Since $\lim_{n\to\infty} \frac{1}{n} = 0$, so the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is convergent. **T** or **F**

    (d) If $0 \le a_n \le b_n$ and $\sum b_n$ is convergent, then $\sum a_n$ converges. **T** or **F**

    (e) $\sum_{n=0}^{\infty} (-\frac{1}{3})^n$ is convergent. **T** or **F**

    (f) $\sum_{n=1}^{\infty} \frac{n}{n^2+3}$ is convergetn. **T** or **F**

    (g) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent. **T** or **F**

11. (10 pts) Prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ is conditional convergent.

Name :________________________________

12. (15 pts) Determine whether the following series is convergent or divergent. Justify your answers.

a) $\sum \frac{\ln n}{n}$.

(b) $\sum \frac{2^n}{n^n}$

(c) $\sum \frac{3^n}{n!}$

13. (10 pts) Find the sum of the series $5 + \frac{10}{5} + \frac{20}{5^2} + \frac{40}{5^3} \cdots$.

14. (10 pts) Find the radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n^3}$

15. (10 pts) Find the Taylor series of $f(x) = \frac{1}{3-x}$ at 2 and its interval of convergence.