MAT 161– Dr. Z. Ye FINAL SPRING 2019

Name :________________________________ ZID :________________________________

Show all work! Don't simplify your results!

1. (10 pts) Let f be defined on an interval (a, b) and $x_0 \in (a, b)$.

   (a) Define the derivative of f at x_0 in terms of a limit of a quotient.

   (b) If $g(x) = -x^2$, find $g'(1)$ by using the **limit definition** of derivative. (Hint, use (a)).

2. (20 pts) Computer the following limits that exist. Justify it if you claim a limit does not exist.

   a) $\lim\limits_{u \to 1} \dfrac{u \sin(\pi u) + 2019 + \ln u}{e^{u-1} - u^3 + 1}$

   b) $\lim\limits_{x \to 2019} \dfrac{|x - 2019|}{x - 2019}$

   c) $\lim\limits_{t \to \infty} \left[\left(\sqrt{t} - \sqrt{t - 4038}\right)\sqrt{t}\right]$

   d) $\lim\limits_{\theta \to 0} \dfrac{1 - \cos\theta}{\theta^2} =$

   e) $\lim\limits_{t \to 1} \dfrac{t^8 - 1}{t^{1/2} - 1} =$

   f) $\lim\limits_{x \to \infty} \dfrac{\ln x}{\sqrt{x}} =$

Name :________________________________

3. (10 pts) (a) Complete the statement of **the Squeeze Theorem** by filling in the following blank spaces. If $f(x) \leq g(x) \leq h(x)$ in an interval $(a-1/2, a+1/2)$, and if

$$\lim_{x\to a} \_\_\_\_\_ = \_\_\_\_\_\_\_ h(x) = A,$$

then

$$\lim_{x\to a} g(x) = \_\_\_\_\_.$$

(b) If $2x^2 - 1 \leq g(x) \leq \sqrt{x}$ for x in the interval $(1-1/2, 1+1/2)$, find $\lim_{x\to 1} g(x)$ by using the Squeeze Theorem.

4. (20 pts) Let

$$y = f(x) = \begin{cases} x^2 - 4 & \text{if } x \leq -2, \\ 0 & \text{if } -2 < x < 2, \\ 1/x & \text{if } 2 \leq x. \end{cases}$$

1) Sketch the graph of the functions $y = f(x)$ and values $f(-2)$ and $f(2)$.

2) Find $\lim_{x\to 2} f(x)$ if it exists. Explain your conclusion if the limit does not exist.

3) Is $f(x)$ continuous at $x = -2$, $x = 0$ or $x = 2$? Give your reasons.

4) Is $f(x)$ differentiable at $x = -2$, or $x = 2$? Explain your conclusions.

5) Is $f(x)$ continuous in intervals $(-3, 0)$ or $(1, 4)$? Give your reasons.

Name :________________________________

5. (15 pts) Differentiate the following functions.

(a) $f(x) = \sqrt{x} + \frac{5}{x} - 2\ln x + 3.$

(b) $g(t) = \frac{t^3 - t - 4}{\sin t - 8}.$

(c) $h(s) = se^{\cos s}.$

6. (5 pts) Let $f(x) = \frac{1}{x}$. Find $f^{(2019)}(x)$.

7. (5 pts) Find $\frac{d}{dx}\int_{x^2}^{0} t\sec t dt.$

8. (10 pts) Let y be a function of x and $xy + \sin y = 2$. Find $\frac{dy}{dx}$ by implicit differentiation.

Name :________________________________

9. (10 pts) A 5-ft ladder is leaning against a wall when its top of the ladder slides down the wall. Bt the time its top is 3 feet from the ground, the top is sliding down at the rate of 2 ft/sec. How fast is the base of the ladder moving away from the wall?

10. (5 pts) Complete the statement of **the Mean Value Theorem** by filling in the following blank spaces. If $f(x)$ is continuous on a closed interval $[a, b]$ and $f(x)$ is differentiable on the open interval (a, b), then there is at least a point x_0 in (a, b) such that

$$f(b) - f(\_\_\_\_\_) = f'(\_\_\_\_\_)(\_\_\_\_\_ - a).$$

11. (10 pts) Let $f(x) = (2x+1)^{-1/3}$ and $f'(x) = -\dfrac{2}{3}(2x+1)^{-4/3}$.

a) Find the slope of the tangent line to the curve $y = f(x)$ at $x = 0$.

b) Find the equation of the tangent line of the curve $y = f(x)$ at $x = 0$.

c) Find the linearization of f at $x = 0$.

d) Use b) to estimate the number $(1.04)^{-1/3}$ and round it to two decimal places. (**No credit if a) is not been used.**)

Name :________________________________

12. (15 pts) Let $f(x) = \dfrac{x^2}{x^2-(100)^2}$, $f'(x) = -\dfrac{2(100)^2 x}{(x^2-(100)^2)^2}$ and $f''(x) = 2(100)^2\dfrac{3x^2+(100)^2}{(x^2-100^2)^3}$.

a) Find the domain of f.

b) Find x-intercepts and y-intercepts.

c) Find all asymptotes (Vertical and Horizontal).

d) Find the intervals on which the function is increasing or decreasing.

e) Identify all function's local extreme (if any), saying where each is assumed and what its value is.

f) Find the intervals on which the function is concave down or concave up. (Hint. You need f''.)

g) Graph the function.

Name :________________________________

13. (10 pts) A box with a square base and closed top must have a volume of $1,000\ cm^3$. Find the dimensions of the box that minimize the amount of material used.

14. (5 pts) Fill in the following blank spaces for a statement of **the Fundamental Theorem of Calculus**. If f is a continuous function on $[a, b]$, then

$$\int_a^b f(x)dx = F(\_\_\_) - F(\_\_\_),$$

where $F(x)$ is an ____________________ of f, that is, $F'(x) =$_____.

15. (20 pts) Find the following integrals:

a) $\displaystyle\int (2\sqrt{x} - \frac{1}{x} + \cos x - 4)dx$

b) $\displaystyle\int_{e^2}^{e} \frac{dx}{x \ln x}$

c) $\displaystyle\int \frac{\sin(2x)}{\cos x} dx$

d) $\displaystyle\int (2x^3 - 2)^2 dx$

e) $\displaystyle\int_{-1}^{2} (e^x - 2|x|)\, dx$

Name :________________________________

16. (5 pts) Find the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i^6}{n^7}-\frac{2}{n}\right)$. (Hint: convert it to a definite integral and evaluate the definite integral.)

17. (5 pts) Set up an integral for the area of the region enclosed by curves $x = y^4$, $y = \sqrt{2-x}$, $y = 0$.

18. (10 pts) Set up an integral for the volume of the solid obtained by rotating the region bounded by curves $y = x^2$, $x = y^2$ about the line $y = 1$.

19. (10 pts) Set up an integral for the volume of the solid obtained by rotating the region (as shown below) bounded by $y = 2x^2 - x^3$ and $y = 0$ about y-axis.